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A Method for Micro-Spectrography of Metals

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Qualitative spectrographic analyses of metallic areas of less than 0.05 mm in diameter can be obtained by means of the equipment and methods described. The apparatus consists of (1) a device for holding a counter electrode and the specimen to be analyzed, (2) an electronic spark generator, and (3) a medium quartz spectrograph. Minute areas such as inclusions, segregations, crystalline phases, surface contaminations, metallic platings, and the extent of metallic diffusion have been successfully analyzed.

SPECTROGRAPHIC analytical methods have been promptly adopted by the metal industries, mainly because of the speed and sensitivity provided. The additional ability of the spectrographic method to produce a characteristic spectrum from a very minute mass of sample suggests a further application in the field of metallurgical analysis; namely, the analysis of microscopic areas on metallurgical specimens.

In our laboratories the need for identifying specific areas such as inclusions, segregates, single crystals, multiconstituent areas, and surface deposits was great enough to warrant the construction of special equipment. The object of this paper is to describe such equipment and some of its applications.

Considerable work has already been done in this field, especially in Germany and England. A great deal of credit should be given to Sheibe and Martin,[1,2] who reported detailed microanalytical techniques as early as 1937. The use of a quartz capillary to restrict the area of specimen sampled and the perfection of a highly polarized intermittent direct current spark were contributions of these men. Thanheiser and Heyes,[3] in 1941, reported the use of drilled holes in thin mica disks for the purpose of confining a spark to a small area of metal. In a series of articles since 1941, Barker, Convey, and Oldfield[4–6] have described the use of pointed graphite and metal counter electrodes for analyzing segregates. The characteristics of the spark perfected by Sheibe and Martin have been investigated in this country by Murray, Gettys, and Ashley.[7]

[1] J. Martin, Dissertation, Technische Hochschule, Munich (1937).
[2] G. Scheibe and J. Martin, "Über eine neue Anwending der Emissionsspektroskopie zur lokalen Mikroanalyse," Spectrochimica Acta **1**, 47–65 (1939).
[3] G. Thanheiser and J. Heyes, "Method of spectrographic analysis of small surface elements," Archiv Eisenhuttenwesen **14**, 543–555 (1941).
[4] F. G. Barker, H. Convey, and J. H. Oldfield, "Spectrographic analysis of segregates," Engineering **152**, 298–300 (1941).
[5] J. Convey and J. H. Oldfield, "Steel segregates," Iron and Steel **18**, 580–584 (1945).
[6] J. Convey and J. H. Oldfield, "Micro-spectrographic analysis of steel segregates," Engineering **160**, 481–484 (1945).
[7] W. M. Murray, B. Gettys, and S. Ashley, J. Opt. Soc. Am. **31**, 433–438 (1941).

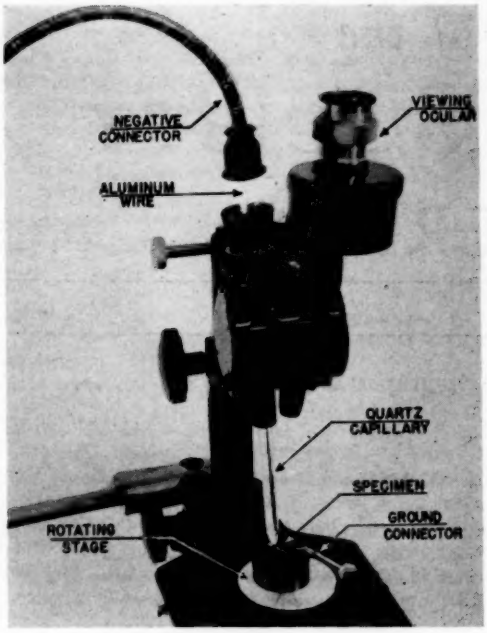


FIG. 1. Binocular microscope converted to provide an adjustable micro-analytical gap between specimen and counter electrode.

APPARATUS

After short preliminary trials with several methods, it was concluded that perhaps the most satisfactory way of greatly restricting the area sampled and yet retaining adequate sensitivity would be the employment of principles determined by Sheibe and Martin. The actual apparatus we have used, however, is of more commonplace construction and is adapted to analyze standard metallographically mounted specimens. This apparatus consists of: (1) a device for holding a counter electrode and the specimen to be analyzed, (2) an electronic spark generator, and (3) a medium quartz spectrograph.

An illustration of a way in which a special counter electrode can confine a spark to an area of less than 0.05 mm in diameter is shown in Fig. 1. A Greenough type binocular microscope is converted to provide an adjustable micro-analytical gap between the specimen and a special counter electrode. One monocular system of the microscope is removed and replaced with a quartz tube with a capillary size bore. The lower end of the tube is ground almost to a point and the very end fused until an opening of only 0.01 mm remains. A fine aluminum wire which acts as the counter electrode is inserted into the capillary to within 0.5 mm of the restricted opening. The mounted specimen is placed on a stage directly beneath the capillary opening so as to provide an analytical gap of about 1 mm. A spring wound clock motor rotates the stage, thus providing a means of moving the specimen while it is being sparked. The remaining microscope ocular facilitates observation and manipulation of the specimen during the sparking process. The entire device for holding the counter electrode and specimen is then mounted directly in front of the entrance slit of the spectrograph.

As can also be seen from Fig. 1, electrons provided through the negative connector are conducted by the aluminum wire to the restricted opening at the end of the capillary. The electrons are then discharged across the 1-mm analytical gap to the specimen, which is grounded. A voltage of 5000 volts is required to jump the gap with regularity, and a current of less than 1 milliampere supplies a satisfactory current density over the minute area covered by the spark.

The electronic spark generator[8] is designed to utilize the properties of a high resistance thermionic valve. This high vacuum tungsten filament tube operates at the high plate voltage and at the low d.c. current output required at the analytical gap. It is quite obvious that the circuit should be polarized so that electrons bombard the specimen, thereby maximizing the radiation emitted by the specimen and minimizing counter electrode radiation. It is also desirable that this direct current be interrupted so as to prevent the thermal destruction of too large an area of sample, and the burning away of the aluminum counter electrode wire. The most desirable way of pulsating the direct current is by means of a condenser in parallel with the analytical gap. The condenser not only provides intermittency but also produces the high initial breakdown currents and the resulting "spark lines" needed for a balance between thermally emitted and collision emitted lines

[8] See references 1, 2, and 7.

The current and voltage supply, of course, should be extremely steady so that the pulse trains are identical with one another.

The complete circuit, as shown in Fig. 2, serves to operate a type 878 control tube in such a way that a precisely controlled voltage, current, and frequency can be supplied across the gap. More specifically, the regulated 110-volt alternating current supplies a variable voltage transformer T1, which in turn furnishes power to the primary of a 5000-volt step-up transformer T2. The high voltage is then rectified by the 866 type tubes, filtered and stored by the condensers C1, and applied to the filament of the 878 type control tube. The 878 tube, acting as a variable high resistance, allows a regulated flow of electrons across the gap and to ground. As to controlling the characteristics of the spark, the variable voltage transformer T1 governs the voltage on the condensers C1 and likewise the voltage applied to the gap. The rheostat R, shown at the top of the diagram, controls the current through the filament of the 878 tube and in turn controls the average current across the gap. The variable air condenser C2 determines the frequency of the discharge and the peak current across the gap.

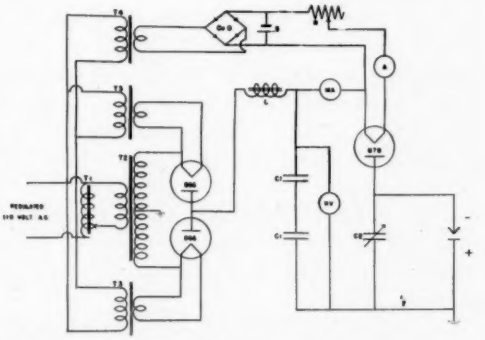
The choice of the spectrograph for this work is based on availability rather than on optical speed. A standard Bausch and Lomb medium quartz instrument is used without alteration of the optics since the same instrument is used for routine analytical work. A constant speed motor is attached through gears to the plate rack-down mechanism in order to produce moving plate spectrograms while the sample passes beneath the spark.

METHODS

In order to localize a spark to a microscopic area of a metallurgical specimen, it is necessary to remove surface irregularities and to provide a mounting which permits the required manipulation of the specimen. Metallographic methods of sectioning, mounting, and polishing yield samples which are very satisfactory for micro-spectrographic analysis. Furthermore, a great many of our analytical problems are also studied microphotographically; thus the same sample preparation can be used for both procedures.

The various types of samples encountered thus far have necessitated the use of two distinct sparking techniques; namely, (1) fixed-sample fixed-plate and (2) moving-sample moving-plate. The first method is used when the analysis requires only the comparison of various separated spots. In such a case, the spark is allowed to burn in each of these spots with the least possible motion of the sample and with no movement of the photographic plate. With this method the exposure time can be increased as much as necessary to produce a sufficiently complete spectrum with only a slight increase in the area of sample covered. Exposures of several hours are sometimes advisable.

Quite often it is necessary to compare the spectrum of a minute area with spectra of the immediately surrounding areas and to determine the extent of diffusion of elements. For this purpose the moving-sample moving-plate method is used. The continuous movement of the sample during sparking and the simultaneous racking down of the photographic plate provide a concentration record of metallic elements from one side of the included area to the other, thus furnishing a comparison of the minute area with



T1 —0–135-volt variable transformer.
T2 —24-ma high voltage transformer, 6000–0–6000.
T3 —2.5-volt, 10-ampere filament transformer with 10,000-volt winding insulation.
T4 —2.5-volt, 5-ampere filament transformer with 10,000-volt winding insulation.
L —40-henry, 2000-ohm, 15-ma smoothing choke.
C1 —4-microfarad, 3000-volt electrolytic capacitor.
C2 —100-micromicrofarad, 10,000-volt variable air capacitor.
CuO—5-ampere, full wave, copper oxide rectifier.
B —2-volt, wet type storage cell.
R —0–.5 ohm, 10-watt rheostat.
MA —0–10, d.c. milliammeter.
KV —0–10,000, d.c. voltmeter.
A —0–5, d.c. ammeter.
– —Cathodic aluminum counter electrode.
+ —Anodic specimen.

FIG. 2. Diagram of the electrical circuit used to produce the micro-spark.

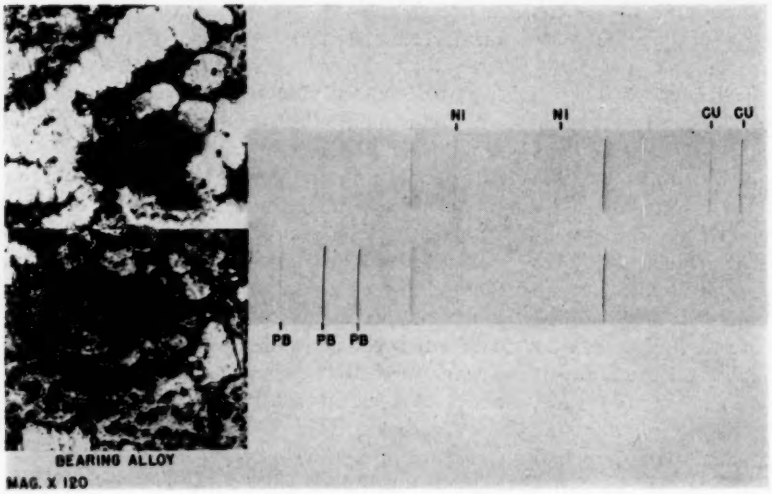


FIG. 3. Spectra of two phases found in a Pb-Cu-Ni bearing alloy. Upper spectrum identifies phase consisting of copper and nickel. Lower spectrum identifies phase consisting of lead.

the surrounding material. In these cases, the rate of motion of the sample must be related to that of the photographic plate in such a way that characteristic spectra can be attributed to each and every microscopic spot along the path of the spark.

The low intensity of light derived from the micro-spark necessitates the extreme conservation of that light. Maximum utilization is effected, of course, when the spark is placed exactly at the entrance slit position or when the spark is focused perfectly at that position. However, in order to retain delineation of the spectra and to keep the spectrograph intact for routine operation, the spark is positioned directly in front of the entrance slit, and no additional lenses are used. This arrangement has provided sufficient line definition and sensitivity for most purposes.

The optical alignment of the spark is accomplished by directing a polychromatic light beam in reverse of the normal direction through the spectrograph so that the light emerges from the entrance slit along the optical axis. The position of the spark gap is then adjusted to coincide with the light beam.

By means of the viewing ocular, the specimen is placed so that the precise area to be sampled is immediately below the counter electrode. The gap distance between the sample surface and the end of the quartz capillary is fixed at approximately 0.5 mm, and the total electrical gap between the sample surface and the end of the aluminum wire is as much as 1.0 mm. The electrical circuit controls are then adjusted to apply a potential of about 5000 volts to the gap. An average current of approximately $\frac{1}{10}$ milliampere and a parallel capacitance of as much as $100\mu\mu$ farads can be increased or decreased depending on the nature of the sample and on the type of spectrum desired. However, an excessive average current or an excessive capacitance tends to destroy both the sample and the counter electrode to an unwanted degree. Although low discharge currents are used, there is nevertheless considerable decomposition of the aluminum counter electrode, causing the gap to lengthen gradually and the capillary opening to congest occasionally.

It has been found that the low light intensity produced by this type of spark ordinarily necessitates photographic exposures of one hour or more. Exposures of several hours may be made by the fixed-sample fixed-plate method in order to reveal minor constituents of an area under test. The moving-sample moving-plate method has been quite satisfactory when the sample speed is adjusted to 2 mm an hour, while the plate racks at the rate of 80 mm during that hour.

Much of the usefulness of either of the two methods depends on the sensitivity of the

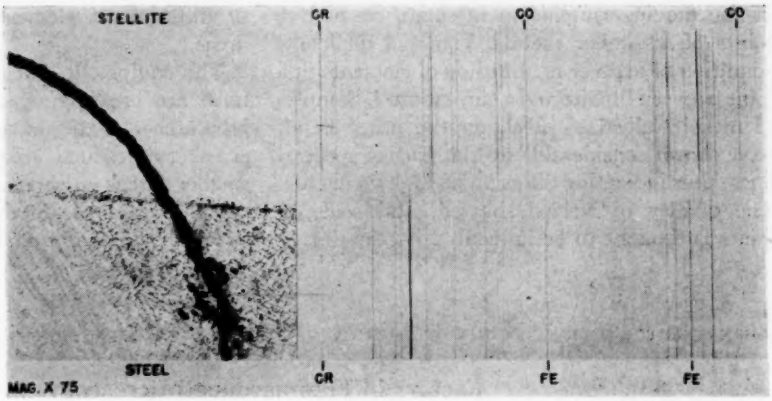


FIG. 4. Spectra obtained by applying the moving-sample moving-plate method to a brazed joint between Stellite and chrome steel. The absence of any appreciable overlapping of spectral lines indicates little or no diffusion of the two alloys.

photographic plates used. Some of the requisites for quantitative spectrography, such as fine grain size and low background density, may be overlooked in order to emphasize higher speed. The substitution of Eastman 103–0 plates for our usual Spectrum Analysis No. 1 plates resulted in considerably greater sensitivity. The 103–0 plate can be developed for a period of 4 or 5 minutes without excessive fog by employing the following developer[9] especially formulated for the reduction of background density:

Solution A	
Sodium sulfite (desiccated)	150 grams
Hydroquinone	20 grams
Water to make	1 liter
Solution B	
Sodium carbonate (desiccated)	50 grams
Potassium bromide	10 grams
Water to make	1 liter

Equal volumes of solutions A and B are combined for use.

The spectra obtained by micro-sparking may be readily interpreted by visual comparison with spark spectra of known materials. When the area to be analyzed consists of an abnormality in an otherwise rather homogeneous specimen of known composition, it is often desirable to have a spectrum of this surrounding normal material on the same plate for comparison. The appearance or disappearance of one or more characteristic lines is often all one needs for a satisfactory qualitative analysis. The smaller the area sparked, the more difficult it becomes to determine composition quantitatively since the spectra are not suitable for photometering and since a reliable internal standard is, of course, absent. In the work carried on by us thus far, no attempt has been made to provide other than qualitative determinations.

[9] Contribution of Dr. R. A. Wolfe, Department of Engineering Research, University of Michigan.

RESULTS

The samples to which we have applied the micro-spark up to this time can generally be assigned to one of the following groups: inclusions, segregations, surface contaminations, crystalline phase identification, diffusion of metals, and metallic platings. In most cases the results obtained have been satisfactory, and typical illustrations of the two sparking methods are given in the following paragraphs.

The determination of the major constituents of one or more phases of an alloy can be exemplified by the analysis of a non-ferrous specimen. This type of result is illustrated in Fig. 3. The particular problem consisted of identifying the elements present in the two conspicuous phases found in a Pb-Cu-Ni bearing alloy. The two phases were sparked independently by the fixed-sample fixed-plate method to provide a separate spectrum for each phase. Interpretation of the spectra reveals that one phase consists of a combination of copper and nickel while the second phase consists of lead.

The determination of the degree of diffusion of metals can be demonstrated by the application

of the moving-sample moving-plate method to joints of dissimilar metals. Figure 4 depicts a condition of little or no diffusion of elements in a joint between Stellite and a chrome steel. Samples of nickel welded to steel, on the other hand, have shown considerable overlapping of spectral lines, thus indicating diffusion of the two metals. The quality of welded, brazed, and soldered joints is thought to be influenced by the degree of diffusion of elements within the adjoining areas.

The equipment and methods in their present form are providing very useful results in this laboratory. Continued development of the micro-spark to produce greater sensitivity for still smaller areas without unduly increasing the complexity of the methods should provide answers to many additional metallurgical problems.

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Factors in Fluorescence Microscopy

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This paper is concerned with a presentation and an attempted evaluation of the various factors governing successful observations with the microscope using fluorescence technics. The primary considerations are: (1) brightness of the fluorescent image in absolute terms and (2) brightness level, image contrast and resolving power of the eye. Details concerning various contributing elements, such as brightness of ultraviolet radiating sources, methods of illumination, selection of optical accessories, and transmitting and absorbing filters for use with visible and ultraviolet radiations, are presented in the text and accompanying tables of data.

IN connection with investigations concerning the fluorescence of some red blood cells, which is discussed in another communication,[1] it became apparent that it was both desirable and necessary to make a critical study of the factors governing successful observations by the fluorescence microscope. In the early stages of the work on the fluorescence of red blood cells it was learned that failure to observe the presence of fluorocytes in the blood was related in all probability to one or the other of the following causes: (1) brightness of the fluorescent image in absolute terms and (2) brightness level, image contrast and the resolving power of the eye.

Although one of us (Hamly)[2] has given some consideration to similar problems with the microscope as ordinarily used, it was felt that these problems were peculiar in the case of the fluorescence microscope.

I. ABSOLUTE BRIGHTNESS

The absolute brightness of the fluorescent microscopic image is limited by the following factors:

1. Brightness of source
2. Method of illumination
   a. Type of source of radiation
   b. Arrangement and focus
   c. Numerical aperture
   d. Transparency of optical ensemble to ultraviolet rays

TABLE I. Partial list of fluorescing standard slides.

Slide number	Common name	Chemical formula and name		Comment
1	Aragonite	$CaCO_3$	Calcium carbonate	Not satisfactory
2	Fluorite	CaF_2	Calcium fluoride	Not satisfactory
4	Willemite	Zn_2SiO_4	Zinc orthosilicate	Very good
6	Uranium nitrate	$U(NO_3)_2$		Not satisfactory
7	Coproporphyrin			Very good
25	Tubercle bacilli (stained with auramine)			Very good

[1] D. H. Hamly, G. J. Hummer, and Charles Sheard, The fluorescence of red blood cells. (Unpublished data.)

[2] D. H. Hamly, "Efficient light sources in photomicrography," Am. Ann. Photography, 58–70 (1943).

3. Selection of filters
   a. Visible red absorptive filters
   b. Ultraviolet transmitting filters
   c. Ultraviolet blocking filters
   d. Ultraviolet identifying mediums
   e. Fluorescing material: influence of coefficient and thickness

The brightness of the final fluorescent image is affected markedly by six factors: (1) the brightness of the source; (2) the transparency of the ultraviolet transmitting filters; (3) the aperture of the illuminating system; (4) the transparency of the optical components to the ultraviolet rays; (5) the efficiency of the ultraviolet-blocking filters in transmitting the region producing fluorescence while, at the same time, absorbing ultraviolet radiation in general, and (6) the thickness and the inherent degree of fluorescence of the specimen.

1. Brightness of Ultraviolet Radiating Sources

Several absolute photometric methods are available for comparing sources used in visible light microscopy but these are difficult to apply to fluorescent images of low brightness. Since evaluation in absolute terms was impractical, twenty-five dummy slides were made for use as a permanent evaluating standard. Some of the materials which were found to be particularly useful are given in Table I. They were dry mounted fluorescing inorganic and organic compounds and balsam mounted stained and unstained tissue sections. The degree of fluorescence varied greatly, for in some cases the material was thin and, in addition, possessed only a low order of fluorescence. Ultimately willemite was used exclusively in source testing, since it had a high order of fluorescence and both large and small crystals were present. While some crystals were visible with every source of ultraviolet radiation tried in these tests, the differences in appearance with variations in intensity of the radiation were so well marked that the selection of the best source was relatively easy. However, coproporphyrin was used in the critical selection of filters, since its blood-red fluorescence was similar to the anticipated fluorescence of the red blood cells.

TABLE II. Sources of ultraviolet radiation.

Kind	Type	Relative intensity	Comment
Cold quartz	Stroblite	Very low	Useless
Tungsten	Spencer special	Low	Useful only for stained material
Mercury arc	Low pressure uviarc	1	Usable
Mercury arc	High pressure AH 4	2	Usable
Carbon arc	Pure, 5 mm, 6–7 amperes	100	Very good
Carbon arc	Sunlight (10 amperes)	100+	Splutters
Carbon arc	*C* (10 amperes)	100+	Splutters
Carbon arc	*E* (10 amperes)	100+	Splutters
Carbon arc	*K* (10 amperes)	100+	Splutters

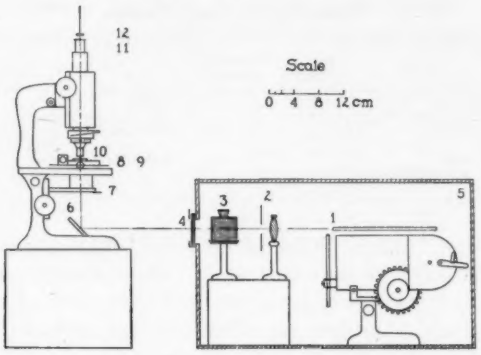

FIG. 1. Ensemble for fluorescence microscopy: (1) carbon arc, 6 amperes with 5 mm+carbon; (2) quartz condenser, 11.5 diopters, with iris diaphragm; (3) cuvette (1-mm glass, or quartz glass, windows) filled with 6.4 percent solution of copper sulfate; (4) Corning ultraviolet transmitting filter; (5) housing for light source and optical accessories; (6) polished aluminum mirror; (7) quartz substage condenser, N.A. 1.25; (8) water film between slide and condenser; (9) material on glass or quartz slide covered with No. 1 coverslip; (10) objective; (11) ocular and (12) Corning ultraviolet absorbing filter.

The standard slides were used in evaluating the relative brightness of the various sources tested. A summary of the results is given in Table II. The arrangement of apparatus used is shown in Fig. 1. All plane surfaces were oriented normal to the optical axis and all lenses were centered carefully and oriented. The Köhler method[3] of focusing and of substage iris control was employed.

2. Method of Illumination

Type of source of radiation, arrangement and focus, numerical aperture and transparency to ultraviolet rays are factors which must be considered.

[3] A. Köhler, "Ein neues Beleuchtungsverfahren für mikrophotographische Zwecke," Zeits. f. wiss. Mikr., Brnschwg. **10**, 433–446 (1893–1894).

TABLE III. Condensers tested.

Kind	Power, diopters	Numerical aperture	Use	Comment
Quartz	11.5		Source	Fairly good
Glass	11.5		Source	Fair
Quartz		*NA* 1.2	Substage	Very good; field small
Glass, 4 mm objective		*NA* 0.66	Substage	Good

a. Type of source of radiation.—One source, namely, the pure carbon arc, when operated at the level of maximal current density, proved to have the requisite high intensity, high uniformity of brightness and high stability when centering and focusing were maintained. The high ultraviolet emitting carbons proved to be too irregular in brightness and too erratic in behavior to be very useful. In addition, the changes in brightness occurred too rapidly for satisfactory adaptation of the eye to take place and hence prevented adequate observation of the material. Although both the AH 4 high pressure lamp and the uviarc quartz mercury arc proved useful in obtaining fluorescence, the approximate relative brightnesses of the willemite and coproporphyrin standard slides to each other was carbon 100, AH 4 2 and uviarc 1 when compared under the standard conditions which were finally adopted.

b. Arrangement and focus.—Figure 1 shows a diagram, drawn to scale, of the fluorescence microscope as finally arranged. The best results were obtained by centering the substage condenser and orienting and centering the source, the source condenser, cuvette and filter on a horizontal line centered on the plane polished aluminum mirror. The carbon arc, operated at maximal current density (6 to 7 amperes for 5-mm positive carbon), was placed so that a clear image, just large enough to fill the aperture of the substage condenser, was formed. The substage condenser then was focused so that the image of the source iris was focused in the plane with the material under observation. The iris was then adjusted in size to just cover the field of view. The only departure from this standard condition stated by Köhler was the use of the substage condenser at full aperture.

c. Aperture.—The information concerning the various condensers which were tested is given in Table III. The double convex source condensers, though possessing considerable spherical aberration at 50-mm aperture, gave good performance at the normal working aperture of 10 mm sufficient for coverage of the 4-mm objective. The superiority of the quartz lens was not great under these circumstances. On the other hand, the quartz substage condenser was very much better than the 4-mm objective when used as recommended by Seggel.[4] It was found, however, that the 4-mm objective gave good optical results when the material was mounted on a cover glass and the illumination was readjusted so that the distance between the objective and the iris of the source condenser was reduced to the mechanical tube length of the objective. Unfortunately, the proximity of the arc to the microscope makes such an arrangement very unsatisfactory. The quartz condenser did not completely cover the desired field when operated at full aperture.

d. Transparency.—As has been indicated by Seggel, the use of quartz is not absolutely essential. In fact it has been noticed that fluorescence could be obtained even though 35 mm of glass were present provided care was taken to obtain the Köhler conditions of illumination. Notwithstanding this, the conditions illustrated in Fig. 1 were found desirable, since work could be carried on without complete dark adaptation. This procedure increased the ease of microscopic manipulation.

3. Selection of filters

There are no filters available at present which transmit ultraviolet radiation and absorb com-

TABLE IV. Some ultraviolet transmitting filters which strongly absorb visible light (used with 39-mm 6.4 percent $CuSO_4 \cdot 5H_2O$).

Number	Name and number	Thickness, mm	Fluorescence brightness with coproporphyrin standard	Background brightness with absorbing filter G34-Y (Corning)
1	Corning 511	6.8	2	3
2	Corning 584	4.9	4	2
3	Corning 585	6.3	9	7
4	Corning 586	11.3	1	0+
8	Corning 986		10	3
9	Schott UG 1	2.5	7	1

[4] K. A. Seggel, "Fluorescenzphänomen und Porphyringehalt der Erythrocyten," Ergeb. d. inn. Med. u. Kinderh. **58**, 582–654 (1940).

pletely the visible light. However, there are varieties of filters, frequently called Wood's filters, which transmit the ultraviolet, violet, and red more or less freely and which absorb strongly the blue, green, and yellow regions of the visible spectrum. These filters, like all others, differ greatly among themselves both in characteristic absorption and in thickness. Considerable technical information[5–10] is available on all filters used in this or similar research with the possible exception of Euphos glass.

a. Visible red absorptive filters.—In German practice 50 mm of a 5 percent solution of $CuSO_4 \cdot 5H_2O$ has been used in a quartz cuvette. In the present investigation 39 mm of a 6.4 percent solution in a container with glass windows 1 mm thick was used as the equivalent. Although there is evidence that more dilute solutions can be used satisfactorily, the stronger solutions should be employed to make certain that no transmission of red occurs. Any such transmission in the red end of the spectrum would be similar to, and therefore possibly mistaken for, a red fluorescence which would be observed by the dark adapted eye.

b. Ultraviolet transmitting filters.—In Table IV are listed ultraviolet transmitting filters which strongly absorb visible light. These were tested under the final standard conditions shown in Fig. 1. The relative brightness of the object and the background is entirely a visual estimate and does not, therefore, indicate true relative brightness. A very dark field was obtained with the Schott glass (filter 9) and the highest contrast was obtained with it. However, Corning 986 (filter 8) proved satisfactory with some illumination in the room. In view of this finding it is probable that a reduction in the thickness of filter 9 (Table IV) would have produced still better results.

TABLE V. Partial list of ultraviolet absorbing-visible transmitting filters.

Name and number	Thickness, mm	Cut off, millimicrons	Fluorescence
Corning G371			4-G
Corning G34-Y	2.1	514 and 730	1-RY*
Corning 243		609 and 715	0
Corning 306	5.9	410 and 730	2-BG†
Euphos	1.1	430 and 720	

* The only one so far tested and found satisfactory with carbon arc.
† Transmitted ultraviolet.

c. Ultraviolet absorbing filters.—In Table V there are listed ultraviolet blocking or absorbing filters and the limits of transmission at the long and short wave-length ends of the spectrum. Ultraviolet absorbing filters were necessary since the bright-field illumination was found to be far superior to the dark-field illumination as obtained by inserting a dark stop in the entrance pupil of the substage condenser. Filter G34-Y was considered best since it transmitted freely the fluorescence of both the coproporphyrin and willemite standard slides and yet fluoresced *per se* relatively little.

d. Ultraviolet identifying filters.—No filter was considered satisfactory which transmitted sufficient ultraviolet radiation to produce fluorescence in the canary yellow uranium glass filter placed above it. This filter also proved valuable in preventing undue exposure of the eye to ultraviolet radiation. This filter was used as a regular check after direct observation of the material was found to produce severe external ocular inflammation and retinal irritation. These symptoms lasted in one instance for more than twenty-four hours.

e. The fluorescing material: influence of coefficient of fluorescence and thickness.—The brightness and distribution of brightness of the fluorescence produced within a solution of fluorescing material varies with the concentration and the thickness of the solution. Also, in the case of a material which is more or less opaque to visible light, the fluorescence will not be as bright when viewed in the direction of the ultraviolet source. These facts explain the bright margins and lower central brightness of thick materials when used under the microscope and

[5] K. S. Gibson and H. J. McNicholas, *Ultraviolet and Visible Transmission of Eye-Protective Glasses* (Bur. Standards, Tech. Papers. No. 119, 1919), 47 pp.
[6] Charles Sheard, *Transmission of Radiant Energy by Ophthalmic Glasses* (Cleveland Press, Chicago, Illinois, 1921), 84 pp.
[7] W. W. Coblentz, and R. Stair, *Spectral-Transmissive Properties and Use of Colored Eye-Protective Glasses* (Circular, Bur. Standards. No. C 421, 1938), 28 pp.
[8] *Glass Color Filters* (Corning Glass Works, Corning, New York), 16 pp.
[9] *Wratten Light Filters* (Eastman Kodak Company, Rochester, New York, 1924), Edition 6, 80 pp.
[10] *Jena Glass-Filter Apparatus* (Jenaer Glaswerk Schott and Gen., Jena, Germany).

the improvement produced when the substage condenser was focused slightly above its normal position.

Furthermore, these observations explain the decreased brightness produced by very thin material and the necessity of employing sources of higher intrinsic brightness to obtain greater fluorescence. In other tests it was found that the use of above-stage illumination frequently gave much greater fluorescence in the case of thick materials as compared with direct transmission of ultraviolet radiation.

II. BRIGHTNESS LEVEL, IMAGE CONTRAST, AND RESOLVING POWER OF THE EYE

Fluorescent images are low in brightness as compared with the images seen in microscopy by the usual methods. It is, therefore, common practice to work in darkened rooms. Conditions of observation become very unfavorable as the size of material decreases. Frequently particles which are easily seen in visible light become invisible with the fluorescence microscope because of low image contrast.

Visual acuity is at a maximal value under yellow light and decreases for green and blue radiations. For any given change in brightness of the test object, the change in visual acuity is the least for monochromatic light. The most recent measurements of the resolving power of the eye show that, under a stable state of adaptation of the eye, the threshold of resolution varied from about 10 or 11 minutes when the background brightness was 6×10^{-6} millilambert to about 0.5 second when it was about 30 to 40 millilamberts. The transition between cone (form and color) vision and rod (light) vision is of the order of 10^{-2} millilambert, but takes place gradually in the range of 10^{-1} to 10^{-2} millilambert. The use of red goggles is recommended if one has occasion to pass into regions of normal lighting.

The ideal conditions are a compromise between high and reduced environmental illumination. Though visibility increases with increasing dark adaptation and critical observations can be made under these conditions, experience has indicated that work under complete darkness should be avoided, since all manipulations are more difficult and because the making and reading of records is virtually impossible.

Also, photomicrography is impractical under conditions of low visibility, for very long exposures may be required to make a record of any sort. While hours may be taken when single photographs are sufficient, the time of exposure must be reduced to seconds when a series of photographs is necessary to demonstrate rapid changes in the material. The effective answer, as is shown in unpublished investigations, is to increase the ultraviolet illumination, for it makes possible photography of fresh animal and plant materials, higher resolving power and the demonstration of the presence of materials of low intrinsic fluorescence.[11]

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[11] R. E. Thiers, E. M. S. Arthur, J. R. Mills, D. H. Hamly, and F. E. Beamish, "Microscopic detection of cadmium oxide particles in lung tissue," J. Indus. Hyg. and Toxicol. (in press).

The Change in Refractive Power of the Human Eye in Dim and Bright Light

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It has been reported that the human eye behaves as though relatively short-sighted in dim light. Observers tend to compensate for this change by setting optical instruments more negatively in dim than in bright light. New measurements of telescope settings by 21 observers reveal an average increase in power of the eye in dim light of 0.59 diopter (range +1.4 diopters to −3.4 diopters). The *dilation of the pupil* in dim light does not contribute significantly to this phenomenon. The *chromatic aberration of the eye* was measured in 14 observers with a specially designed *spectral stigmatoscope*. The refractive power of the eye increases about 3.2 diopters between 750 and 365 mμ. For this reason the Purkinje shift of maximum visual sensitivity from 560 mμ in bright light to 505 mμ in dim light produces a relative myopia in dim light of 0.35 to 0.40 diopter. Persons who display changes larger or smaller than this do so because of involuntary changes in the *accommodation* in bright and dim light. In dim light the eye enters a state of relatively *fixed focus*, little different from its condition when the accommodation is paralyzed with homatropine. In this fixed state the accommodation may be relaxed, or it may add as much as 3 diopters to the refractive power of the eye. Experienced observers focus optical instruments in dim light close to the optimal settings determined objectively. Departures of more than 0.5 diopter in either direction from the optimal focus depress the visual sensitivity and acuity. It is concluded that setting optical instruments about 0.4 diopter more negatively in dim than in bright light is justified on the basis of the chromatic aberration of the eye. Many observers gain a further advantage from individual adjustments of focus in dim light, appropriate to their accommodative behavior.

DURING the late war it was reported from a number of American and European sources that the human eye behaves as though relatively short-sighted (myopic) in dim light, and that one can compensate for its increased refractive power with a negative lens correction. It has been stated, for example, that the vision of normal persons is improved at night by the use of spectacles with −1 to −2 diopter lenses[1]; and that persons who are hyperopic by 1 to 2 diopters can improve their night vision by putting aside their spectacles, while myopes are in general better off in dim light using their lens corrections. Similarly, it has been reported that observers tend to focus optical devices more negatively in dim than in bright light.[2]

The present paper is concerned with an examination of this phenomenon, its magnitude, and its causes. Among the latter, three primary factors must be considered:

(1) Changes associated with the opening of the pupil in dim light, involving the spherical aberration of the eye, and perhaps other, less well defined, phenomena.

(2) Changes associated with the Purkinje shift of spectral sensitivity toward lower wavelengths in dim light, in conjunction with the chromatic aberration of the eye.

(3) Possible differences in the state of accommodation of the eye in dim and bright light.

I. MAGNITUDE OF THE EFFECT

Our measurements are based in large part on the focusing of field glasses in dim and bright light, since the investigation originated in a request from the armed services for this type of information.

Many persons are known to adjust the focus of *monocular* devices negatively even in bright light. This tendency appears to be due to unconscious accommodation of the eye, associated with the effort to achieve the best visual acuity. The observer compensates for his accommodation by an appropriate negative setting of the instrument. It is commonly believed that this tendency

* Work done under contract with the Office of Scientific Research and Development. A preliminary account of this investigation was presented at the meeting of the Optical Society in March, 1946 (Griffin and Wald, 1946). We should like to acknowledge the assistance of Ruth Hubbard with the preliminary experiments.

[1] The diopter, a unit of refractive power, is the reciprocal of the focal length in meters.

[2] We have had access to restricted reports on this subject from American, British, and French military and governmental sources. Unfortunately these have not to our knowledge been published, and so are not available for specific reference. The reports themselves are on file with the Army-Navy-N.R.C. Vision Committee, Natural Science Building, University of Michigan, Ann Arbor.

is more prominent in inexperienced persons, and that with long use observers relax the accommodation and go over to more positive focal settings.

It has been found in practice that monocular settings in bright light tend to be distributed widely—over a range of about 4 to 5 diopters—and that the average settings are about 1 to 2 diopters more negative than the ordinary refractive corrections of the observers. For this reason fixed-focus instruments frequently have built into them corrections of -0.5 to -1.5 diopters.

With *binocular* instruments the only available information indicates that focal settings in bright light approximate the normal refractive corrections of the observers. In this case there appears to be no tendency to set instruments negatively. The reason is almost certainly that the convergence of the eyes normally is closely tied to their accommodation, and so restrains the degree of accommodation which can be achieved without double vision or misalignment with the optic axes of the instrument.

In both binocular and monocular situations, the transfer from bright light to dim moonlight or starlight illuminations is said to involve a special negativity of setting, estimated by various investigators at about 1 to 2 diopters.

To redetermine the magnitude of this change, the focusing of binoculars was studied in observers who made repeated settings, using the same instruments, outdoors and in an indoor range, at daylight and starlight levels of illumination.

Three types of binoculars were used: (1) the standard U. S. Navy 7×50 glasses, (2) the 7×50 glasses with oculars so modified as to convert them to 5×50, and (3) a pair similarly converted to 10×50. The latter two instruments required supplementary positive lenses over the oculars in order that targets at 42 feet—our indoor distance—could be brought within the available range of focusing adjustment.

Outdoors these instruments were used both mounted and hand-held. In the laboratory they were always mounted and the observers were provided with a chin-and-forehead rest to minimize head movements. In all outdoor work the observers set the inter-pupillary distance of the binoculars themselves. In the laboratory we accurately aligned the optic axes of the binoculars with each observer's pupils.

What we shall call a "trial" consisted of a series of five to ten settings of the binoculars in bright light, and a corresponding series in dim light, with other conditions held as nearly constant as possible. The final result of each such trial was a value for the difference between the average settings in bright and dim light. In all, 110 trials were conducted with 21 observers who ranged in age from 17 to 30 years.

Each individual setting was made in the following manner: One objective lens was covered, and the ocular of the open side was turned to its most positive setting. The observer was told to look through the instrument and to adjust the focus until he obtained the clearest definition of the target. He was permitted to pass back and forth through the area of best focus, but was discouraged from taking more than about one minute to make each setting. This process was repeated with the other objective covered. Finally the observer was asked to look through the instrument with both lenses uncovered, and to make any further adjustments he wished. Only rarely did subjects make any further changes, so that the settings reported here are primarily monocular, although they were checked with both eyes and therefore also were acceptable binocularly.

The net result of all trials was an average setting in dim light 0.54 diopter more negative than in bright light.[3] The differences in individual pairs of trials were wide, ranging from -3.4 diopters to $+1.4$ diopters (dim light minus bright light settings), so that no very precise significance can be attached to the average. Thirty-six trials involved differences of at least -1 diopter; and 9 of these involved differences of at least -2 diopters. Of the occasional groups of readings which averaged more positive in dim than in bright light, in only 6 trials were the differences as great as 1 diopter. In general the positive settings tended to occur with the less experienced observers.[4]

[3] As will appear below, the chromatic aberration of the binoculars themselves introduces a change in focus of about $+0.05$ diopter in going from bright to dim light. For this reason a more correct value of the average increase in power of the *naked* eye in dim light is 0.59 diopter.

[4] In an earlier discussion of these experiments (Griffin and Wald, 1946), we included a pair of positive differences

TABLE I. Changes in focus setting in bright and dim light. Data of 8 observers, using binoculars of various types under various conditions. The table lists the differences between average settings in bright and in dim light in diopters. A minus sign indicates a shift to more negative settings in dim light.

Binoculars:		10×50	5×50	5×50	5×50	7×50	7×50	
Conditions of use:		Mounted indoors	Mounted indoors	Mounted outdoors	Hand-held outdoors	Mounted outdoors	Hand-held outdoors	
Observer	Eye	Change in focus setting (diopters)						Averages
I. W.	L.	−2.2	−1.3	−1.1		0.1		−1.1
	R.	−3.4	−0.5	−0.8		1.1		−1.0
D. R.	L.	−0.4	−1.0	1.4		1.1		0.3
	R.	0.0	−1.0	0.1		−0.4		−0.3
V. P.	L.	0.3	−0.3			−0.1		0.0
	R.	−0.8	−0.5			0.2		−0.4
J. McA.	L.		−0.5	−0.2		−1.2		−0.6
	R.		−0.7	0.0		0.0		−0.2
F. S.	L.	0.6		0.6	0.7	0.2	−0.3	0.4
	R.	−0.9		0.1	0.1	0.7	−0.6	−0.1
H. J.	L.	0.9		−1.4	−0.8	−0.3	−0.1	−0.3
	R.	0.7		−0.1	0.2	−0.6	−0.7	−0.1
A. G.	L.	−2.9		−1.0	0.2	−0.5	−1.9	−1.2
	R.	−2.3		−1.2	−1.2	−0.3	−1.3	−1.2
R. B.	L.	−0.7		−0.6	−0.4	0.2	−1.1	−0.5
	R.	0.3		−1.0	−0.8	−0.3	−0.9	−0.5
							Over-all average	−0.43

Several observers who had set the instruments much more negatively in dim than in bright light were immediately offered their bright light settings and were asked to compare the two. About half of them reported that the daylight setting gave them as good vision as did the more negative setting which they had themselves selected. The others definitely preferred their more negative settings.

An idea of the variability of these settings can be gained from Table I. Eight of our more experienced observers had made settings in bright and dim light, indoors and out, with three instruments both mounted and hand-held. Table I shows for each observer the average differences between settings in bright and dim light. There was considerable variation between individuals, and in the same individual using the same or different instruments. Occasionally there was even as much as 1.5 diopters difference between the two eyes in a single trial. There is, however, a clear trend toward more negative settings in dim light. The average difference in Table I is −0.43 diopter, to be compared with our average difference of −0.54 diopter in all trials.

II. FACTORS ASSOCIATED WITH AN EXPANDED PUPIL

A. General Considerations

Among possible causes of the apparent myopia of the eye in dim light it is necessary to consider the effects of dilation of the pupil. As this brings the outer zones of the cornea and lens into the refracting system of the eye, it could introduce significant changes in focal length.

A spherical lens of finite thickness exhibits spherical aberration: its marginal portions possess a shorter focal length than its center. Consequently, even in monochromatic light, the image of an object point is spread out for some distance along the optic axis. On stopping the lens down

in dim light (+1.9 and +2.2 diopters in the two eyes) yielded by an observer who participated in only one trial; and another pair of first settings by another observer (+2.6 and +2.7 diopters) who in a second trial no longer yielded positive settings. These two pairs of unconfirmed first settings, which as it happens also represented the most positive differences in setting in dim light in all our observations, have been discarded in the present report.

so that only its center is employed, one approaches a true point-for-point focus, and incidentally the longest focal length of which the lens is capable.

If the human eye exhibits aberration of this type, it should be relatively hyperopic in bright light, in which the pupil is smallest, and should grow increasingly myopic as the pupil opens in dim light. Such a relative myopia would induce a compensatory negative setting of optical instruments in dim light.

This phenomenon would be enhanced by the Stiles-Crawford effect. According to this, under conditions of cone vision—in bright light or in foveal fields—light passing through central portions of the pupil are much more effective visually than light passing through peripheral zones. In effect this factor aids the pupil in stopping down daylight vision to the central region of the refractive system. In dim lights, however, in which vision is governed by rods, the Stiles-Crawford effect is no longer observed. Thus as the pupil expands in dim light the full refractive effects of its marginal zones should be realized (Crawford, 1937–38; Stiles, 1939).

Actually the average human eye is reasonably well corrected for spherical aberration. The cornea has a flatter curvature, and hence refracts light less strongly at its periphery than at its center. More important still is the fact that the center of the lens is composed of more highly refractive material than its peripheral portions. Both circumstances tend to counteract the effects of spherical aberration. The central area is usually somewhat undercorrected, and so displays a small positive aberration, while the peripheral zones are usually somewhat overcorrected. In the average eye the net spherical aberration is small, much smaller for example than the chromatic aberration.[5]

Ames and Proctor (1921) have measured in detail the focal lengths of various zones of the eye in two observers. Moving outward from the optic axis, the eye at first grows increasingly myopic. This change reaches a maximum at about 1.5 mm from the center of the pupil—its approximate daylight radius. From here outward the tendency is usually reversed: the eye becomes increasingly *hyperopic*.* In some instances there is very little further change in focus. In going from a day to a night pupil, therefore, there is either negligible change in focal length of the eye, or more usually a slight *lengthening* of focus. Such changes as this cannot contribute to the relative myopia of the eye in dim light, unless to counteract it slightly.

B. Observations

It is however possible that the opening of the pupil involves other, more obscure factors than the spherical aberration. In such a situation as this an empirical demonstration is more convincing than any amount of theoretical argument. The following tests provide direct evidence that within the limits of accuracy of our observations the opening of the pupil has negligible effects on the focus:

(1) In a short series of measurements in dim light, two skilled observers compared the distances at which a point of light was in best focus, using the natural pupil and the pupil stopped down with a diaphragm to a diameter of 2 mm. No consistent differences could be observed.

(2) In the course of a special group of experiments (see below; Figs. 4–6), three subjects focused a telescope in bright light with the normal contracted pupils, and again after the pupils had been maximally expanded with homatropine. The settings were not significantly different in the two instances.

(3) When binoculars are focused upon bright stars at night—an operation which ordinarily involves foveal cone vision—neither we nor others have observed consistent differences from daylight settings. Since in such star settings the pupil is expanded, one must conclude that this factor in itself introduces no change in focus.

(4) A series of measurements was made in the laboratory with 7×50 binoculars, using natural

[5] Hartridge (1941), p. 412; Duke-Elder (1944), p. 771. Hartridge comments, "We may say, therefore, that in everyday life the effects of spherical aberration are altogether negligible, compared with those of diffraction and chromatic aberration." This was also the conclusion of Matthiessen (1847), one of the earliest workers to attempt to measure the chromatic aberration of the eye accurately: "L'oeil humain est moin achromatique encore que l'on ne pensait; mais, en revanche, il est exempt d'aberration de courbure pour la lumière comprise entre *B* et *G* et pour des faisceaux peu obliques, pénétrant à travers une pupille peu dilatée; . . . ."

* Cf. also Gullstrand (1909), p. 429.

pupils, and as target the capillary arc of a high pressure mercury lamp, viewed through a filter which transmitted only the 492-mμ line. This monochromatic source eliminates any possible effects of chromatic aberration, and is in a region of the spectrum which readily stimulates either rods or cones. Against a background illumination of 200–300 millilamberts, the arc appeared as a thin blue-green line. These are photopic conditions, involving central, cone vision. Settings were made by 13 observers under such conditions; and again under conditions of off-center, rod vision at a very low level of room illumination, and with the arc so dimmed that it appeared colorless, and could not be seen at all when fixated centrally. Though individual observers yielded considerable differences in setting under both circumstances, the average difference of setting in dim light was only about −0.1 diopter. The very different pupillary apertures in the two cases, combined with the change from cone to rod vision and from central to off-center fixation, had produced an almost negligible difference in the average focus.

These experiments exclude, as important in changing the focus, not only effects within the eye which might be supposed to accompany the opening of the pupil, but a number of external complications as well. One of these is the spherical aberration of the binoculars, for the exit pupils of the instruments varied between 7 and 10 mm in diameter, and were therefore at least as large as the dilated pupils of the eyes. The opening of the latter in dim light therefore simultaneously opened the effective pupils of the binoculars, without appreciable effect on the focus. Another complication so excluded is the possibility of systematic misalignment of the eyes with the binoculars when used in dim light, since this should have appeared as prominently in the present trials as in our main body of observations.

Finally these experiments indicate certain special situations in which the transfer from bright to dim light does not involve important changes in the average power of the eye—e.g., when observations are confined to the fovea, or when they are made in monochromatic light. The special significance of these conditions is that they eliminate the effects of the chromatic aberration of the eye on the focus.

III. THE CHROMATIC ABERRATION OF THE EYE

A. General Considerations

As the eye goes from bright to dim light, its relative sensitivity to the various portions of the spectrum shifts toward lower wave-lengths. This is the Purkinje phenomenon. In very dim light, in which vision is governed by rods, the maximum sensitivity is at about 505 mμ. In bright light in which vision is dominated by cones, the maximum sensitivity varies between about 550 mμ in the peripheral retina, and 560–565 mμ in the foveal region; these differences are due to the presence of yellow pigmentation in the central retina (Wald, 1945a). Since the Purkinje shift depends specifically upon transfers between cone and rod vision, the spectral sensitivity of the fovea, which contains only cones, remains unchanged whatever the illumination.

An optical system which depends upon refraction for forming an image ordinarily exhibits chromatic aberration: it refracts light more strongly, and hence brings it to a shorter focus, the shorter the wave-length. Therefore in mixed radiation the image of an object point is focused in the form of a spectrum along the optic axis, the violet rays at the shortest distance, the red at the longest distance.

That the human eye exhibits chromatic aberration has been known since Newton (1730). Measurements of the difference in focus of the eye in the red and violet were made by Thomas Young (1802), Fraunhofer (1817), Matthiessen (1847), and Helmholtz (1866), all of whom estimated it to be between 1.5 and 2 diopters. Some shortening of focus should therefore accompany the transition from cone to rod vision, from bright to dim light.

In recent years, more extensive measurements of the chromatic aberration of the eye have been made by Nutting (1914), Ames and Proctor (1921), Polack (1923), and Pinegin (1941). The results are in only approximate agreement with one another, and each of these researches involved only from 1 to 9 observers. In the most extensive investigation, that of Pinegin, no indication whatever is given of the methods used.

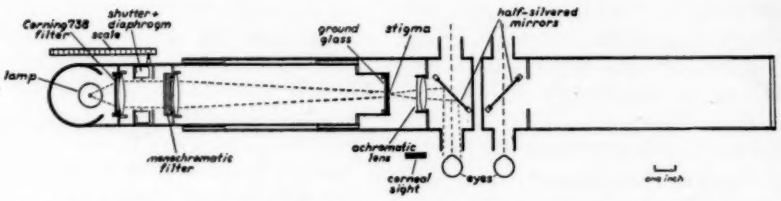

FIG. 1. Sectional view of spectral stigmatoscope. The lamp, filters, shutter, iris, and light point or "stigma" are mounted in a single housing which the observer moves back and forth by means of a rack and pinion until the stigma appears in sharp focus. Accommodation is stabilized by looking directly through the instrument at an external target (not shown). The light source is a G. E. H-3 mercury arc, from which the short ultraviolet radiation is screened with a Corning 738 filter, and spectral lines and narrow bands are isolated with monochromatic filters. The cornea of the observer's eye is set at the principal focus of the achromatic lens by means of a movable head rest, which is adjusted so that the vertex of the cornea is in line with the corneal sight shown in the figure.

For all these reasons we thought it advisable to redetermine the chromatic aberration of the eye with a reasonably large number of observers.

B. Methods: A Spectral Stigmatoscope

The chromatic aberration of the eye was measured with a specially designed spectral stigmatoscope. The plan of this instrument is shown in Fig. 1. In essence it involves a movable point of monochromatic light of various wave-lengths—the "stigma"—the position of which is adjusted by the observer so as to bring it into best focus.

In the present instrument the cornea of the observer's eye is placed at the principal focus of an achromatic lens of 10.42-cm focal length.[6] With such an arrangement, the distance from the lens to the stigma at best focus is directly proportional to the aberration. The instrument is provided with a scale to read directly the value of the aberration in diopters.[7]

The unique feature of this instrument is that the source of light for the stigma is a high pressure mercury arc (the General Electric H-3 lamp), from the radiation of which nine spectral lines or narrow monochromatic bands, ranging from 365 to 750 mμ, are isolated with specially selected color filters (Wald, 1945b). A photographic shutter is built into the lamp housing so that the light can be presented either continuously or in flashes. The intensity at each wave-length can be controlled with an iris diaphragm, mounted in a collimated portion of the light path.

It is essential in making stigmatoscopic observations that the eye maintain a steady state of accommodation. This is achieved best when both eyes are used; the accommodation is then reinforced by the associated convergence. For this reason, though the stigmatoscopic measurements involved only one eye, the accommodation was fixed binocularly. Semi-transparent mirrors were mounted so that the observer saw the stigma by reflection from one mirror while looking through both mirrors at a target 30 feet away.

The target was a pattern of black stripes 1.3 cm wide on a white background, with a black, 8-cm square at its center. It was centered on the optic axis of the stigmatoscope. The observer's eyes were brought into such alignment with the instrument that the stigma appeared as though projected on the black square. The stigma therefore was truly monochromatic; no light was added to it by the external target. When sharply focused, the stigma subtended a visual angle of about 13 minutes.

The measurements were performed in a dark-room, with the target illuminated so that its white portions were at a brightness of 0.1 millilambert. The intensity of the stigma was adjusted with neutral filters and the iris diaphragm so that at most wave-lengths it was about 1.5 logarithmic units above the visual threshold. The mercury lamp did not provide sufficient energy for this at 365 or 750 mμ; but the stigma was still bright enough at these wave-lengths for all the observers to make consistent settings.

[6] The focal length of this lens was found to vary less than 0.05 cm between 405 and 750 mμ.

[7] We are much indebted to Dr. Kenneth Ogle of the Dartmouth Eye Institute for advice on the general design and use of stigmatoscopes.

Actually, within broad limits, the brightness of the stigma seemed unimportant; exploratory measurements at one wave-length showed that from the visual threshold to 3 logarithmic units above threshold, focus settings remained practically independent of the intensity.

In making measurements the observer was instructed to concentrate his attention on the external target, the details of which were only barely visible at the level of illumination employed. The observer varied the distance of the stigma until it appeared with maximum distinctness. With most subjects a series of settings at one wave-length fell within a range of 0.5 diopter. Occasionally a considerably more myopic setting was made, apparently because the subject had neglected the target and had accommodated to some degree for near vision. When this happened, the observer was cautioned to concentrate on the target more carefully, almost always with the result that the settings returned to the previous range.

C. Observations

With this procedure, reliable data were obtained from 14 observers. The average settings for each observer are presented in Table II, and the average for all observers is shown in Fig. 2.

Chromatic aberration can be expressed in various ways: the change with wave-length of the focal length of the eye, or of its reciprocal, the power of the eye. We have preferred to deal with these data in their most useful form—the correction in diopters which must be added to the eye to adjust its focus at various wave-lengths. That is, with the eye in best focus at 578 mμ, the reference wave-length we have chosen, our data express the diopter correction which brings it into correct focus at other wave-lengths. It should be borne in mind that the more negative this correction, the shorter is the focal length and the greater is the power of the eye.

Our measurements are in reasonably good agreement with those of previous workers, with which they are compared in Fig. 3. In this figure, the data of other workers have been transposed vertically so as to come together with our own at 578 mμ. This is a valid procedure, since it involves only a change of the reference wave-length, at which the aberration is considered to be zero.

It is interesting to note that a large fraction of the chromatic aberration of the eye can be ascribed to dispersion by water. Helmholtz had calculated the difference in focal length for red and violet light in a hypothetical eye composed of water, and concluded: ". . . so far as chromatic

TABLE II. *Axial chromatic aberration of the human eye.* Measurements on 14 observers, with the spectral stigmatoscope. The axial aberration is expressed as the lens correction in diopters required at each wave-length to bring the eye to the same power it possesses at 578 mμ. The actual settings for each observer at 578 mμ are shown in the fourth column; all other settings are stated relative to these. The "water eye" is a structure of the same posterior focal length as the human eye, but entirely composed of distilled water, and possessing a single refracting surface.

Observer	Age	Sex	Actual correction at 578 mμ	Axial aberration 365 mμ	405 mμ	436 mμ	492 mμ	546 mμ	578 mμ	621 mμ	691 mμ	750 mμ
T. C.	18	M	−0.46	−2.88	−1.94	−1.38	−0.70	−0.16	0.00	0.22	0.41	0.62
D. G.	30	M	−3.31	−2.70	−1.86	−1.36	−0.77	−0.18	0.00	0.26	0.45	0.61
E. F.	21	F	−1.38	−2.08	−1.27	−0.74	−0.50	−0.31	0.00	0.13	0.50	0.40
T. E.	17	M	−0.27	−2.61	−1.53	−1.12	−0.51	−0.27	0.00	0.15	0.46	0.56
R. M.	17	M	−0.21	−2.45	−1.62	−1.22	−0.83	−0.53	0.00	0.09	0.49	0.58
C. P.	24	M	−1.49	−2.93	−2.04	−1.23	−0.63	−0.12	0.00	0.38	0.54	0.91
F. O.	18	M	−0.43	−3.27	−1.55	−0.89	−0.39	−0.05	0.00	0.21	0.35	0.67
N. W.	17	M	−1.74	−2.52	−1.67	−1.23	−0.58	−0.16	0.00	0.15	0.50	0.62
T. H.	18	M	−2.09	−2.74	−1.68	−1.25	−0.42	0.00	0.00	0.12	0.23	0.54
R. S.	25	M	−0.65	−2.51	−1.80	−1.31	−0.62	−0.10	0.00	0.14	0.52	0.73
G. B.	26	M	+3.52	−2.50	−1.65	−1.17	−0.68	−0.08	0.00	0.15	0.53	0.65
M. L.	19	F	−0.17	−1.96	−1.57	−0.98	−0.35	−0.11	0.00	0.09	0.18	0.54
A. Y.	21	F	−0.19	−3.00	−1.85	−1.28	−0.55	−0.14	0.00	0.20	0.53	0.68
S. E.	19	F	−1.13	−2.65	−1.79	−1.45	−0.66	−0.25	0.00	0.25	0.46	0.60
Averages			−0.71	−2.63	−1.70	−1.19	−0.59	−0.16	0.00	0.18	0.40	0.62
"Water eye"			*394mμ*	*436mμ*	*468mμ*	*486mμ*	*534mμ*	*546mμ*	*579mμ*	*589mμ*	*656mμ*	*671mμ*	*708mμ*
Axial aberration			−1.353	−0.910	−0.627	−0.497	−0.214	−0.145	0.000	0.046	0.290	0.336	0.589

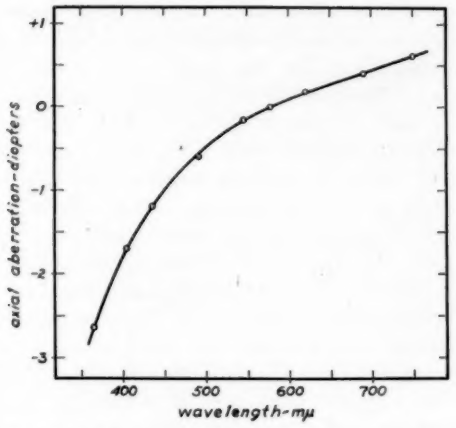

FIG. 2. Axial chromatic aberration of the human eye. Averages of measurements on 14 observers. The ordinates show the lens correction in diopters needed to give each eye the same refractive power that it possesses when accommodated for distance vision at 578 mμ.

aberration is concerned, the human eye corresponds very closely to an eye of distilled water; the dispersion of the natural eye being if anything a little higher. Perhaps, we might conjecture that, just as the refractivity of the crystalline lens is greater than that of pure water, so also its dispersion is higher."*

We have made a comparable calculation to determine what proportion of the observed chromatic aberration of the eye can be ascribed to dispersion of light by water alone. For this calculation we have considered an eye of the same posterior focal length as the normal human eye (22.875 mm, according to Gullstrand), entirely composed of distilled water, and possessing a single spherical refracting surface. This surface must be assumed to have a radius of 5.719 mm, very close, therefore, to the radius of curvature of the reduced eye of Gullstrand (5.73 mm). For the index of refraction of water at various wave-lengths, we used data in the *International Critical Tables*.[8]

The calculated chromatic aberration of such a "water eye" is shown in Table II and Fig. 3. It is of the same general form, but is definitely smaller than the observed chromatic aberration of the human eye. Between 578 and 436 mμ, the "water eye" possesses an aberration of 0.91 diopter. The average aberration which we observed over this wave-length range is 1.19 diopters, while Pinegin's data over the same range average 1.5 diopters.

We have also made a few rough measurements with the Zeiss Pulfrich Refractometer of the indices of refraction, at 436 and 546 mμ, of the aqueous and vitreous humors of cattle eyes. These fluids, as one might expect from their chemical composition, possess refractions and dispersions much like those of distilled water. No reliable data on the cornea were obtained; since it is very thin, however, and its outer and inner surfaces are not far from concentric, it cannot make a large contribution to the total dispersion of the eye. A crude attempt was made also to estimate the dispersion of the cattle lens. The measurements were not reliable since the lens substance was compressed and distorted, but they did indicate a considerably higher dispersion than that of water.

We later learned of the measurements of Kunst (1895) on the dispersion of human ocular media, which lead to about the same result. Kunst found the mean differences in refractive index between 486 and 589 mμ to be as follows: cornea, 0.00375; aqueous humor, 0.0041; lens cortex, 0.00535; lens nucleus, 0.0060. (The dispersion of water between the same wave-lengths is 0.00414.) There is good reason to believe, therefore, as Helmholtz had supposed, that the departure of the observed chromatic aberration of the eye from that of a "water eye" is principally caused by higher dispersion by the lens.

D. Effects of Chromatic Aberration on the Focus

In going from foveal cone to peripheral rod vision, the maximum sensitivity of the eye shifts from about 560 to 505 mμ. This change in wave-length corresponds in our measurements to an increase in power of the eye of about 0.35 to 0.40 diopter. Any change in the energy distribution of the source as one goes from bright to dim light would of course modify this result. Apart from such special complications, however, one may expect the eye to become 0.35 to 0.40 diopter more myopic in going from foveal cone to rod vision on the basis of the chromatic aberration of the eye alone.

It might be asked what the chromatic aberra-

* Cf. 1924, p. 174.
[8] Volume 7, p. 13.

tion of the binoculars contributes to observations with these instruments. To check this we have made measurements at various wave-lengths with the standard 7×50 binoculars, setting the oculars so as to bring the image of the source projected onto a screen into sharpest focus. Taking the average setting at 578 mμ as reference point, as in the case of the eye, we found the settings at 546 mμ to be displaced −0.04 diopter, at 492 mμ +0.08 diopter, and at 436 mμ +0.42 diopter. It can be estimated from these measurements that the change from 560 to 505 mμ associated with the Purkinje shift would introduce a change of setting of the binoculars of about +0.05 diopter. This value, which is so small as to be almost insignificant in our observations, is opposite in direction from the observed change in setting of the binoculars in dim light. It may therefore be added to the observed average (0.54 diopter) to give a more correct value for the change in power of the naked eye in dim light, which then becomes 0.59 diopter.

To this total change the chromatic aberration of the eye contributes only 0.35–0.40 diopter. It will be recalled also that many observers regularly make settings in dim light much more negative than the average, sometimes insisting that they see better with settings as much as 3.4 diopters more negative than their daylight settings. Also, although rarely, a consistent series of settings has been obtained which is more positive in dim than in bright light. It is clear that some highly individual factor, in addition to chromatic aberration, must operate in these cases. The most plausible assumption is that this is accommodation.

IV. ACCOMMODATION OF THE EYE IN DIM LIGHT

A. General Considerations

By accommodation is meant the increase in power of the eye associated with the thickening of the lens. The lens is stretched thin in the relaxed eye by the tension on the suspensory ligament (the zonule of Zinn). In accommodation the contraction of the annular fibers of the ciliary muscle relieves this tension, and permits the lens to become more spherical through elastic contraction of its capsule. In observers 20 to 30 years of age the power of the eye can be varied in this way some 7 to 11 diopters. The ability to accommodate declines steadily with age, however, and at 60 is usually reduced to about 1 diopter.

Under ordinary conditions, accommodation is a wholly involuntary adjustment, the point of which is to bring the retinal image of the viewed object into the optimal condition for visual resolution. When both eyes are used, this adjustment is reflexly reinforced by convergence—the alignment of the eyes so that the images of the fixated object fall upon both foveas.

In very dim light these ordinary devices for fixing the accommodation must deteriorate greatly. With the decline of visual acuity at low illuminations the retina is less able to evaluate the sharpness of the image thrown upon it, and hence to guide the accommodation. In lights too dim to stimulate the foveas of both eyes, furthermore, the primary mechanism of con-

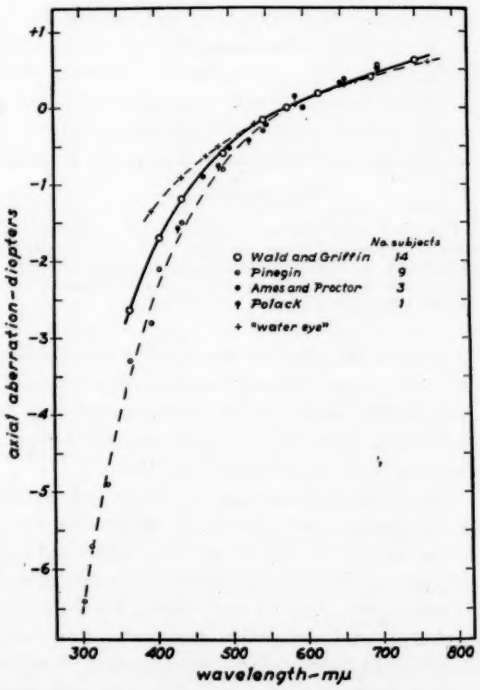

FIG. 3. Axial chromatic aberration of the human eye. The present measurements compared with those of previous observers and with the calculated chromatic aberration of a hypothetical eye composed entirely of water. Ordinates as in Fig. 2.

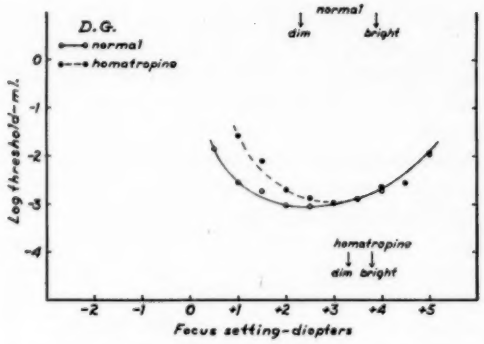

FIG. 4. Log brightness needed to achieve a constant visual acuity at various focus settings of a 7×50 telescope. Observer D. G. Accommodation paralyzed with 2 drops of 3 percent homatropine; near point, 1 meter. Observer's own focus settings in dim and bright light for the normal and homatropinized eye are indicated with arrows.

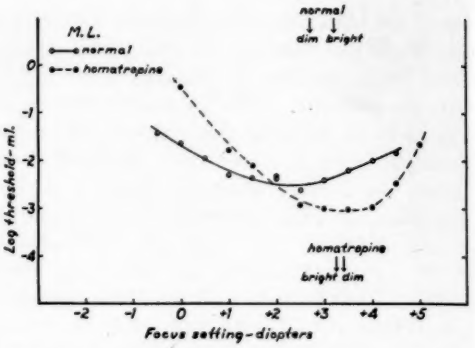

FIG. 6. Log brightness thresholds for constant visual acuity at various focus settings of a 7×50 telescope. Observer M. L. Accommodation paralyzed with 2 drops of 3 percent homatropine; near point, 64 cm. Observer's own focus settings are indicated with arrows.

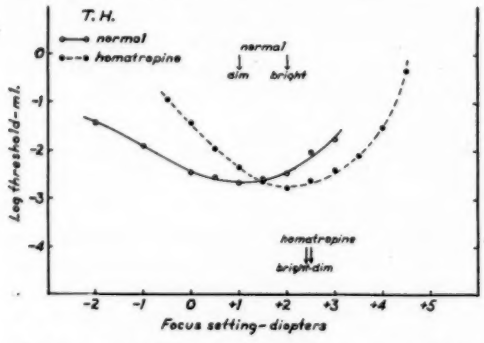

FIG. 5. Log brightness thresholds for constant visual acuity at various focus settings of a 7×50 telescope. Observer T. H. Accommodation paralyzed with 2 drops of 3 percent homatropine; near point, 60 cm. Observer's own focus settings are indicated with arrows.

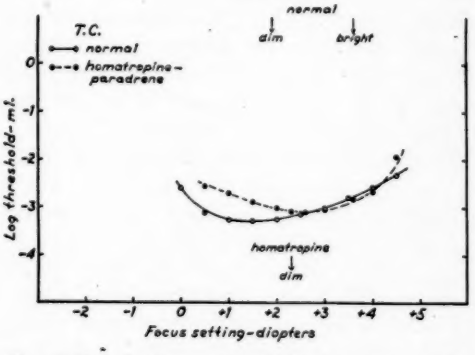

FIG. 7. Log brightness thresholds for constant visual acuity at various focus settings of a 7×50 telescope. Observer T. C. Accommodation paralyzed with 3 drops of 1 percent paradrene plus 2 drops of 3 percent homatropine; near point, 1.5 to 2 meters. Observer's own focus settings are indicated with arrows.

vergence must cease to operate. For these reasons one may expect the adjustment of accommodation in dim light to be very coarse; and the possibility exists that it fails entirely either in the relaxed or some other condition.

These uncertainties are increased by the use of an optical instrument of adjustable focus. When such a device is used in bright light, as indicated above, the eye is free to accommodate to any degree, and to compensate for its accommodation by appropriate settings of the instrument. With monocular instruments a considerable accommodation combined with compensatory negative settings are frequently observed; while with binocular instruments the tendency to accommodate appears to be restrained by the associated convergence, which would throw the eyes out of alignment with the axes of the instrument. In very dim lights the stimuli, both for accommodation and for its restraint by convergence, are so greatly weakened that it is difficult to anticipate what behavior to expect.

For these reasons we arranged an experiment which was intended to give some objective estimate of the state of accommodation in dim light, and of its capacity to adapt to imposed displacements of the focus. As a measure of the effectiveness of focus we used the threshold brightness at which a single visual acuity target, viewed through a telescope, could just be resolved. Such thresholds were measured for a

series of arbitrary ocular settings in the normal eye. The measurements were repeated with the accommodation paralyzed with homatropine. The subjects were also permitted to make their own settings on the same target in bright and dim light and with normal and paralyzed accommodation.

The experiments were performed with five subjects, specially selected because they set binoculars more negatively in dim light than could be accounted for by the chromatic aberration. In these observers, therefore, there was at least presumptive evidence of increased accommodation in dim light.

B. Procedure

As visual acuity target, alternating black and white stripes were employed, 0.9 cm in width, covering a disk 21 cm in diameter. The disk was mounted on a pivot, so that the orientation of the stripes could be altered by the experimenter. The subject used one side of a pair of standard 7×50 binoculars, mounted on a tripod 42 feet from the target, and his head was supported and eye aligned as described above. The target was illuminated with white tungsten radiation (color temperature about 3200°K), the brightness of which could be varied continuously with a neutral optical wedge.

In these experiments all observations were monocular. The focus of the telescope was arbitrarily set at various values over a range of at least 5 diopters. At each such setting, the brightness of the target was first set well below the threshold for seeing the stripes, and was raised slowly until the subject could just determine the direction in which the stripes were oriented. At the optimal focus setting the stripes could be recognized at a brightness of about 0.001 millilambert, well within the range of rod vision.

The results of this group of observations are shown in Figs. 4 to 8. Various aspects of the measurements bear upon distinct though related issues, which for greater clarity are developed separately below.

C. Fixed Power of the Eye in Dim Light

In Figs. 4 to 8, the open circles and solid lines show the way in which change of setting of the telescope affects the visual acuity threshold in the normal eye.

One should expect a young observer to be able to accommodate for displacement of focus of an optical instrument, so as to maintain his visual acuity unimpaired over a range of settings of several diopters. This is in fact the behavior observed when such an experiment is performed

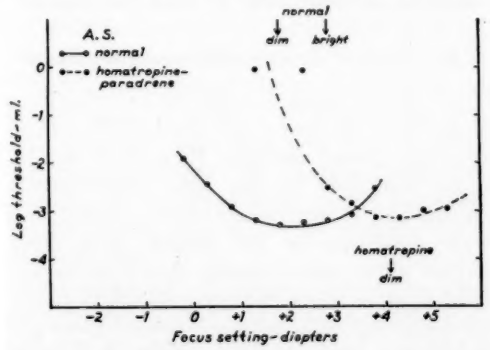

FIG. 8. Log brightness thresholds for constant visual acuity at various focus settings of a 7×50 telescope. Observer A. S. Accommodation paralyzed with 3 drops of 1 percent paradrene plus 1½ drops of 3 percent homatropine; near point greater than 1 meter. Observer's own focus settings are indicated with arrows. This is a hyperopic subject who accommodates habitually to achieve a high visual acuity without spectacles.

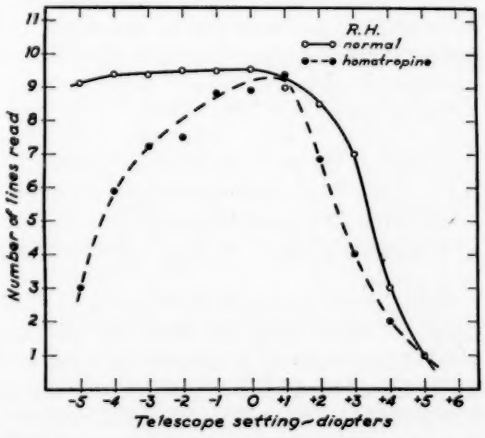

FIG. 9. Effect of off-setting of a telescope on the visual acuity in bright daylight. Ordinates: number of lines of a Snellen chart read at 290 feet. When the telescope is offset in the negative direction, the normal eye maintains its visual acuity by compensatory accommodation. After treatment with homatropine (4 drops 5 percent) this capacity is lost, and the visual acuity declines sharply with off-setting of the focus in either direction.

in bright light (see below, Fig. 9). In very dim light a different result is obtained. Each of the solid curves of Figs. 4 to 8 passes through a surprisingly sharp minimum, representing the optimal setting for visual resolution. At as little as 0.5 to 1 diopter to either side of this setting, appreciably higher brightnesses are needed for resolving the target.

It appears from these measurements that in very dim light the eye has almost lost the capacity to adapt its power to the addition of external lenses. It seems to be in a condition of relatively fixed focus.

This inference is confirmed when one repeats the same experiment with the accommodation paralyzed in the relaxed condition with homatropine, or homatropine and paradrene (solid circles and broken lines of Figs. 4 to 8). The legend of each figure states the dosage of drug employed, together with the near point of the eye, determined either immediately before or after the measurements. That the near point in all cases was at least 60 cm from the eye shows that the range of adaptive accommodation which remained was no more than 1.7 diopters, and in most observers 1 diopter or less.

If the normal eye in dim light were able to accommodate adaptively, one should expect homatropinization to make it respond much more sharply to offsetting of the focus. The broken curves of Figs. 4 to 8 do show a small tendency in this direction, a slight narrowing compared with the results obtained with normal eyes. But much more impressive is the great similarity of the data obtained under both circumstances. They show clearly that the normal eye in dim light is capable of only slightly greater accommodation than an eye paralyzed with homatropine.

The force of this argument becomes apparent when the results just described are compared with observations of a similar type made in bright light. At high brightnesses one expects that a subject who starts with the accommodation relaxed should be unable to compensate for the insertion of positive lenses before the eye, but should be able to adapt by accommodation for a wide range of negative lenses. This is the behavior actually observed.

We have made several series of trials in bright daylight, using one eye and one side of the 7×50 binoculars, and as target a Snellen chart at a distance of 290 feet. The chart was read at a series of arbitrary settings of the telescope, and the number of lines read was taken as a rough measure of the visual acuity. Figure 9 (solid line) shows the result for one observer, 22 years of age. The acuity declines sharply as settings are made more positive, but it is maintained almost unaltered from zero setting to −5 diopter, the limit of the instrument. A hyperopic observer, aged 39, in a similar test was able to compensate for offsetting of the instrument about 3 diopters in either direction from zero, apparently because his initial state was one of partial accommodation.

Under these circumstances, paralyzing the accommodation with homatropine has a very large effect (Fig. 9, broken line). Now the same visual acuity is achieved at zero setting as with the normal eye; but the acuity declines sharply for displacements of setting in either direction, yielding a function very much like that obtained with either the homatropinized or the normal eye in dim light.

It can be concluded with some assurance that in very dim light—specifically under conditions of rod vision—the eye is in a state of relatively fixed focus. It can adapt its accommodation only slightly if at all to the addition of external lenses. More important, it must be equally restricted in its ability to adjust to differences in the distances of objects viewed with the naked eye.

D. Effect of Accommodation on the Focus

It will be recalled that the experiments of Figs. 4 to 8 were performed with subjects specially chosen because they set binoculars more negatively in dim light than could be ascribed to chromatic aberration alone. In all these observers, as the figures show, homatropine shifts the minimum threshold—and indeed most or all of the entire function relating threshold to ocular setting—toward more positive settings. It is clear from this that the power of the eye is decreased by homatropine; that is, that in dim light all these subjects had been in a state of partial accommodation. The degree of accommodation which is revealed by homatropinization is of about the right magnitude to

account for the special negativity of setting of these observers.

To check this conclusion these subjects were allowed to set the telescope themselves on the same target, in dim and bright light, with the normal eye and under the influence of homatropine. The averages of these measurements are entered in Figs. 4 to 8 as small arrows. With normal eyes the average difference of setting in dim and bright light was −1.2 diopters. The two observers who had been treated with both homatropine and paradrene found the brightly illuminated target viewed through the telescope so unpleasant that they could not make satisfactory settings. The others (Figs. 4 to 6) after treatment with homatropine yielded differences of setting in dim and bright light of −0.5, +0.1, and +0.15 diopter, average −0.1 diopter. Paralysis of the accommodation had entirely abolished their special negativity of setting.[9]

The settings shown in Figs. 4 to 6 reveal another significant datum. The settings in bright light are *not* appreciably altered by homatropine. It must be concluded that in bright light these subjects maintained relaxed accommodations. It was specifically low illuminations that called forth the fixed, partly contracted state which we have described.

The observer of Fig. 8 requires special consideration. This was a hyperopic subject, who was accustomed to achieve normal visual acuity without spectacles by accommodating. It is clear that his telescope settings in bright light were made with the eye partly accommodated; and for this reason, unlike the other subjects, he yielded much more positive settings, even in dim light, after treatment with homatropine. With the normal eye in dim light this observer accommodated still further, as shown by the difference of 1.0 diopter between his normal settings in dim and bright light.

The experiments of Figs. 4 to 8 provide added assurance that the opening of the pupil has little effect on the power of the eye. The pupil is dilated maximally by homatropine, yet hardly more than the dilation of the normal pupil in dim light, so that the large effects of homatropine at low illuminations cannot be ascribed to this factor. It was noted that paradrene alone, which at first has little effect on accommodation while producing a maximum pupillary dilation, does not during this interval change appreciably the visual acuity threshold in dim light. Finally, the observations in bright light show no significant difference in settings made with the normal eye, in which the pupil is highly contracted, and with the maximally expanded pupil of the homatropinized eye.

It can be concluded that persons who set optical instruments more negatively in dim light, than can be accounted for by the chromatic aberration of the eye, do so because of a fixed state of partial accommodation. Conversely most of our observed differences of setting in dim and bright light, since they could be accounted for by the chromatic aberration alone, apparently were made with the accommodation relaxed. Finally, a hyperopic observer who habitually accommodated even in bright light (cf. Fig. 8), could, by relaxing his accommodation somewhat in dim light, yield more *positive* settings than at high illuminations. This is the probable explanation of the small number of instances in which we observed this type of behavior.

E. Efficiency of Focusing

With the eye in a state of fixed accommodation in dim light, any auxiliary optical device must bear the entire burden of adjusting the focus to the visual task. Under these circumstances it is important to know the reliability with which subjects adjust the focus of such an instrument as a telescope in dim light.

The data of Figs. 4 to 8 provide some indication of this. The minima of the curves for the normal and the homatropinized eye constitute the optimal settings for visual acuity. In every instance the subject's own settings in dim light lie very close to these objective optima.

All these subjects were college students, already reasonably experienced in this type of observation. It may be concluded at least for this type of observer that his own average settings are nearly optimal.

[9] We would expect a more extensive series of settings by homatropinized subjects to yield an average difference of setting in bright and dim light close to the −0.30 to −0.35 diopter accounted for by the combined chromatic aberrations of the eye and of this type of telescope.

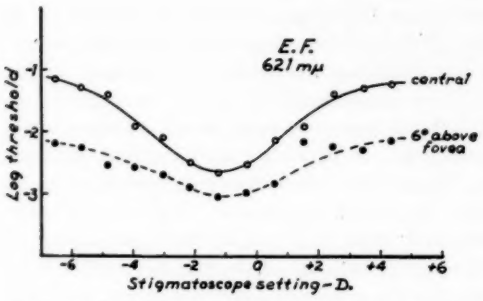

FIG. 10. Log brightness for just seeing a small source of monochromatic light when in and out of focus. Wave-length 621 mμ. Abscissae in diopters, as though the source were at a fixed distance, and lenses of the stated powers were placed before the eye. The measurements show that the visual threshold for a small source of light is minimal when it is in exact focus; and that displacements of focus raise the threshold much more in the fovea than in the peripheral retina.

F. Latitude of Focus for Pattern Vision and Light Perception

We have assumed that one factor leading to the failure of adaptive accommodation in dim light is the low resolving power of the retina, which is for this reason less able to evaluate the sharpness of the image thrown upon it. An out-of-focus image is, however, not simply coarsened in pattern, but has also suffered losses in brightness contrast. In view of these considerations it may be asked how important sharpness of focus is in dim light, and specifically how much the focus may be displaced without visual detriment.

The data of Figs. 4 to 8 (solid lines) include such information. They show that, at a fixed distance, optimal pattern vision in dim light is obtained when the eye is combined with a specific power of external lens. Displacement of setting of 0.5 diopter in either direction from the optimum demanded in most cases an appreciably higher brightness for resolving the target. Off-setting the focus by as much as 1 diopter in either direction caused significant increases in threshold brightness in all the observers, averaging 0.21 logarithmic unit or about 60 percent.

The discussion to this point has involved the conditions for optimal pattern vision. If one inquires instead into the conditions for simple detection of light, comparable results are obtained.

We have made such measurements with the spectral stigmatoscope. The experimental arrangements were much the same as those described above in connection with our measurements of chromatic aberration. The stigma was seen as though projected upon a black square at the center of a striped target at a distance of 30 feet. For foveal observations the white portions of the target were set at a brightness of 0.1 millilambert, and the eye was fixated upon the black square. For peripheral observations a dim red fixation point was provided, placed some distance above the striped target. In this way the image of the stigma was made to fall either within the fovea or 6° above the fovea. The former condition involves cone vision, the latter rod vision.

Such measurements were performed with two observers at two wave-lengths, 546 and 621 mμ. The observer set the stigma at its best focus at each wave-length; then the stigma was displaced in either direction so as to be out of focus. At each setting the brightness was determined at which the stigma was just visible. The results are shown in Figs. 10 and 11.

In each instance the brightness threshold passes through a distinct minimum, which corresponds to the position of most accurate focus. In both the fovea and the peripheral retina this is the condition in which a small field of light is most effective as a stimulus.

The figures show also that displacement of focus has a much greater effect in the fovea than in the periphery. This result was to be expected. When sharply focused, the stigma subtends a visual angle of about 13 minutes. When it is out of focus, its radiation is diffused over a wider retinal area. We have found in an independent investigation that the total luminous flux required to stimulate peripheral rod vision rises very gradually as the diameter of the test field is increased from about 15 minutes to about 1.5°; while the flux needed to stimulate the fovea rises very rapidly over the same range of field sizes. (Wald, Hubbard and Griffin, 1945.)[10]

V. CONCLUSIONS

We have found the refractive power of the human eye to increase in dim light by an average

[10] The data of Graham and Bartlett (1939), recomputed in terms of total light required to stimulate vision (brightness×area), lead to the same conclusion.

value of about 0.6 diopter, as measured by the compensatory negative setting of telescopic instruments. In a large proportion of trials the increase in power was much greater than this. To a first approximation, factors associated with the opening of the pupil do not contribute to these changes. Because of the large chromatic aberration of the eye, the Purkinje shift of spectral luminosity in going from cone to rod vision produces an increase in power of 0.35 to 0.40 diopter. Those persons who display changes which depart in either direction from this value do so because of systematic, involuntary differences in accommodation in dim and bright light.

This analysis clarifies a number of special situations in which the transfer from bright to dim light involves little or no change in the power of the eye.

Observations wholly restricted to the fovea are of this type. Since the fovea contains only cones, it does not display the Purkinje phenomenon; hence the chromatic aberration of the eye does not affect the focus for light of constant spectral composition. The only changes to be expected in this case are those associated with accommodation. It is possible, moreover, that as long as foveal vision is maintained, the eye continues to accommodate efficiently to the visual task. In this event no change in power is expected on going from bright to dim light. It is significant that such a common foveal operation as setting a telescope upon a bright star at night yields results scarcely to be distinguished from daylight settings.

Observations in monochromatic light, whether or not they include a transfer from cone to rod vision, do not involve appreciable changes in the power of the eye unless a change in accommodation has occurred. The chromatic aberration is of no consequence in this instance. So, for example, the experiment cited in Section II above, in which settings were made upon the 492 mμ radiation of the mercury arc, yielded an average difference in setting in dim and bright light of only −0.1 diopter. The data of Figs. 10 and 11 provide another instance of this relation. The focus settings which yield minimum thresholds in monochromatic light do not change in the transfer from foveal cone to peripheral rod vision.

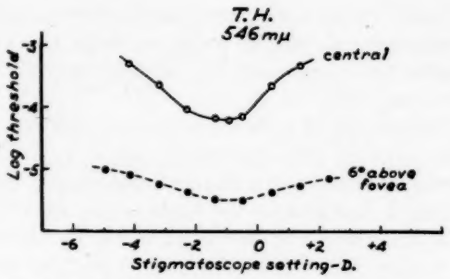

FIG. 11. Log brightness for just seeing a small source of monochromatic light when in and out of focus. Wavelength 546 mμ. Otherwise as in Fig. 10.

Finally, if changes in accommodation in bright and dim light are prevented, only effects associated with the chromatic aberration of the eye remain. Figures 4 to 6 provide examples of this type of restriction. Paralysis of the accommodation with homatropine in these experiments reduces the change in focus in bright and dim light from an average value of −1.0 diopter to one of −0.1 diopter.

Probably the most significant observation made in the course of these experiments involves the relatively fixed state of accommodation of the scotopic eye. As we have noted, one would in any case have expected the regulation of accommodation in dim light to become very coarse. What happens, however, appears to be much more drastic than a simple loss of precision. It amounts to an almost complete failure of adaptive accommodation, a condition resembling that of the eye paralyzed with homatropine. The relatively fixed state in dim light may range from the completely relaxed condition found in some observers, to accommodations of the order of 2 to 3 diopters.

The eye in this condition is set for vision at a restricted range of distances, which in emmetropic observers with relaxed accommodation stretches from the far point to infinity, but in other persons may draw in as close as 33 cm (3 diopters accommodation). It should be possible to demonstrate experimentally such optimal ranges for pattern vision in dim light, varying greatly from one individual to another.

Under these circumstances an auxiliary optical instrument must take over the entire function of adjusting the focus to the visual task. If set at optimal focus, it can greatly improve the

scotopic visual acuity and sensitivity to light. Conversely, if out of focus, it finds the eye unable to compensate by accommodation for its discrepancy.

The setting of optical instruments for use in dim light is thus peculiarly critical. A flat change of about −0.4 diopter from the daylight setting is justified on the basis of the Purkinje shift and chromatic aberration of the eye alone. It is true that the setting can depart by this amount from the optimum without serious detriment. Numerous observers will therefore suffer no large impairment of vision using their bright light settings at all illuminations. Nevertheless a real advantage should in general follow from scotopic settings about 0.4 diopter more negative than the daylight settings. If the question of a fixed displacement of focus for general use in dim light should arise, we should give first consideration to a change of about this size, perhaps between this and the average change in focus we have observed—say 0.5 diopter.

The reasonably large number of observers in whom the power of the eye changes more than 0.5 diopter in dim light are very considerably aided by being permitted to make individual settings. So, for example, the subjects who yielded the data of Figs. 4 to 8 needed on the average only half the intensity of light at their optimal settings in dim light, that they would have required had they retained their daylight settings.

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Optical Silver Chloride*

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(Received January 6, 1947)

Optical silver chloride in the form of plates, sheet, film, and special shapes has been developed to meet both wartime and peacetime needs for this unique infra-red transmitting material. The remarkable plasticity of silver chloride has been used to develop the technique of forming this material into various optical shapes not possible with other crystalline optics. Large single crystals are grown by the gradient method. Such single crystals are more easily worked into various shapes than is possible by using the multiple cast ingot as a starting point. Optical and transmission data as determined up-to-date is given.

SILVER chloride has long been known to possess many unique physical properties not common to other halide salts. Upon fusion it forms a wax-like horny mass which wets both glass and metal surfaces and in this connection has long been used for making seals of glass to metals.

Reinkober[1] reported that solid silver chloride up to 3 mm thick would transmit 50 percent radiation up to 22 microns. Reports however on the possible use of silver chloride plates and films for infra-red transmitting materials are very meagre. Randall[2] reported in 1939 that silver chloride could be rolled into sheet.

Fugassi and McKinney[3] made the first intensive study of the possibility of producing silver chloride sheet of sufficient size to be utilized as cell windows for infra-red spectroscopy.

In the present development of optical silver chloride sheet and film, methods were devised whereby production was stepped up to meet demands for large commercial quantities of the material for infra-red work.

PREPARATION OF SILVER CHLORIDE

Ordinary so-called reagent grade silver chloride as produced for the chemical trade was found to be entirely unsuited for producing high grade silver chloride film. Such factors as raw materials used, protection from light and the presence of very small amounts of impurities serve to contribute to the problem of obtaining satisfactory film. The following general procedure gives best results.

Commercial ingot silver is dissolved in C. P. nitric acid and the resulting solution evaporated to a sufficient concentration to allow the silver nitrate to crystallize out. The crystals are washed and dried centrifugally. The so-called first crop of nitrate crystals constitutes the raw material for producing the chloride. C. P. hydrochloric acid is used for precipitating the silver chloride and the product is thoroughly washed with distilled water.

The freshly prepared silver chloride is fused in Pyrex ware and either cast into ingot for storage or loaded directly into the crystallizing furnace for the production of single crystal ingots.

The production of silver chloride and all subsequent operations to the finished sheet are carried out only in artificial light. Nothing stronger than a tungsten incandescent light can be used.

PRODUCTION OF INGOTS FOR ROLLING

In the early stages of this development the molten silver chloride was cast into rectangular ingots in nickel moulds. Prior to rolling, the ingots were planed down to remove dross and cloudy portions due to dissolved gases liberated during cooling of the ingot. At least 50 percent of the ingot thus was lost before the rolling operation. Such ingots consisted of a mass of small crystallites of silver chloride of varying size depending upon the rate of cooling. In the subsequent rolling of the ingot the crystallites of silver chloride are sufficiently plastic to enable them to flow over each other by a mechanism of slip

* Presented at the Cleveland Meeting of the Optical Society of America.
[1] O. Reinkober, *Handbuch und Jahrbuch der Chemischen Physik* (1934), Vol. 9, p. 39.
[2] J. T. Randall, Trans. Faraday Soc. **35**, 9 (1939).
[3] P. Fugassi and D. S. McKinney, Rev. Sci. Inst. **13**, 55 (1942).

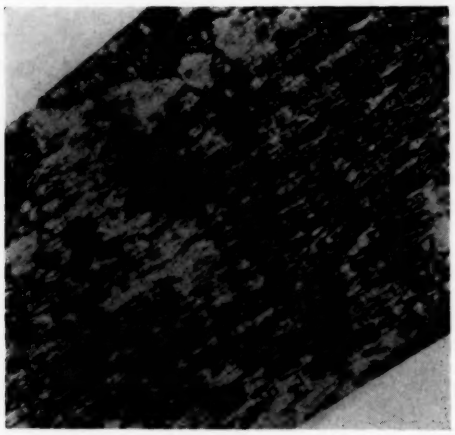

FIG. 1. Silver chloride sheet from multiple crystal viewed between crossed Polaroids.

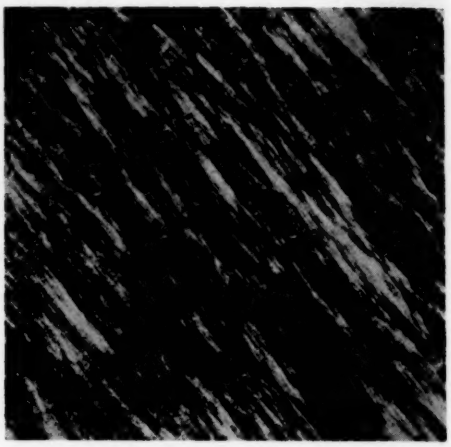

FIG. 2. Silver chloride sheet from single crystal viewed between crossed Polaroids.

banding or lattice deformation. A transparent continuous plastic sheet results.

In common with the experience of Fugassi and McKinney[3] it was found that aging of the ingots rendered them much more susceptible to cracking upon rolling. The ingots were heated to 200°C before rolling or pressing and during the reduction process frequent reannealing to 200° was necessary. From a thickness of .075″ on down rolling could proceed without further reheating, provided there was no intermediate storage of a partially rolled sheet. In such an event careful reannealing had to be resorted to before further rolling could proceed.

SINGLE CRYSTAL INGOTS

It was found that if the molten halide were allowed to solidify slowly in the ingot larger crystals were formed. Likewise it was found that the subsequent rolling of such an ingot proceeded much better. This suggested that single crystals would be ideal for rolling. Accordingly, the technique for growing large single crystals by the elevator gradient method[4] similar to that used for the production of optical crystals of rocksalt, potassium bromide, etc., was worked out.

Pyrex vessels are used as crucibles and by using a four unit furnace, sixteen ten-pound silver chloride crystals are grown simultaneously. The crystals are invariably single and completely transparent. The single crystal ingots can be stored indefinitely without any physical change taking place.

PROCESSING OF INGOTS

Single crystal ingots can be turned on a lathe, planed on a shaper, sawn or subjected to any of the operations common to plastic materials. Although the crystal is cubic in habit it is impossible to cleave along the usual cleavage plane.

These ingots in contrast to multiple crystal ingots show a remarkable range of plasticity. In the processing for production of sheet and film, an ingot 3.5 inches in diameter, when preheated to 400°C is reduced to a flat billet 1 inch thick in one pressing operation. From this point on the billet is immediately rolled to sheet without further reheating or annealing. Reduction of the billet is approximately 10 percent of its thickness per pass through the rolls. Sheet and film with an optical surface is produced by rolling between highly polished nickel and chrome plated steel sheets.

A comparison of transmission photographs of sheet produced from single crystal and that produced from multiple crystal ingots taken between crossed Polaroids clearly indicates the non-homogeneous pattern of the multiple crystal sheet in Figs. 1 and 2. It is furthermore found

[4] H. C. Kremers, Ind. Eng. Chem. **32**, 1478 (1940).

that multiple crystal sheet becomes embrittled upon aging. In single crystal sheet this tendency is entirely lacking.

Various optical shapes (see Fig. 3) such as plates, curved windows, cone shaped windows, lenses, prisms, etc., can readily be formed by impact flow pressing. For cold pressing of such optics, pressures of approximately 15,000 pounds per square inch are required. It is also essential that the crystal blank used in the pressing operation approximate somewhat the final shape desired. Special shapes such as rods and tubes can be extruded by corresponding pressures.

Lap welds of sheet are readily made by pressing the overlapped seam between plattens of a hydraulic press at a temperature from 180°–200°C. Flame welding of joints in the sheet or joining sheet to glass, noble metals or silver plated base metal can be carried out with the usual gas-air or gas-oxygen mixtures.

CORROSIVE PROPERTIES OF SILVER CHLORIDE

In common with the halides, silver chloride is very corrosive in contact with base metals. Noble metal halides being inherently unstable, the halogen is readily lost either by heat or in contact with a metal higher in the electromotive series. At elevated temperatures such metals as zinc, cadmium, iron, brass, aluminum, and copper are vigorously attacked. It has been the experience of the author to have the optical surface of a hard steel pressing die completely ruined by one cold pressing of a silver chloride shape.

Alloys such as stainless steel, monel, Hastalloy, Nichrome, etc., are more resistant. Nickel is resistant enough so that it may be used as moulds for ingots, plates for hot pressing or for rolling the film. Noble metals such as gold, silver, and platinum are entirely inert to silver chloride.

For general impact formation of finished optical parts hardened steel dies optically finished and protected with chrome plate are satisfactory. Such dies can be used for several hundred pressings without appreciable injury to the optical surface.

PROPERTIES OF OPTICAL SILVER CHLORIDE

Optical silver chloride as prepared from single crystal ingots is completely transparent and colorless provided extreme care is used in preparing pure silver chloride.

The melting point of silver chloride was redetermined and found to be 457.5°C (reported value is 455°C).

Silver chloride sheet is not effected by light from an incandescent tungsten bulb but will slowly change color when exposed to fluorescent light or diffused daylight. Bright sunlight will darken the material fairly rapidly. It has also been found that the tendency of sheet to darken is in direct relation to the extent of impurities present.

The index of refraction of silver chloride is 2.065 for the ND sodium vapor line.[5] The hardness is 2.5.

The transmission of silver chloride in the infrared has been determined by a number of workers. Fugassi and McKinney[3] report transmission data on films prepared from multiple crystal ingots. They found that transmission varies with thickness. Sheet of 1.20 mm thickness showing a maximum transmission of some 35 percent. By measuring sheets of varying thickness their values calculated to zero thickness gives 74 percent as the transmission left after losses due to surface reflection and scattering.

Transmission of silver chloride sheet prepared from single crystal ingot is shown in Fig. 4. The value of 80 percent is in agreement with the theoretical value as calculated by Fuoss[6] using the index n AgCl = 2.06 for silver chloride.

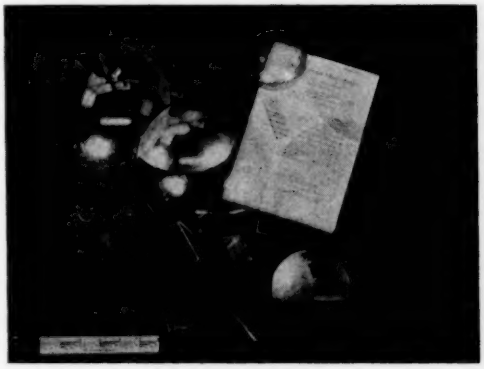

FIG. 3. Silver chloride windows and pressed lenses.

[5] L. Wernicke, Pogg. Ann. **142**, 560 (1871).
[6] R. M. Fuoss, Rev. Sci. Inst. **16**, 154 (1945).

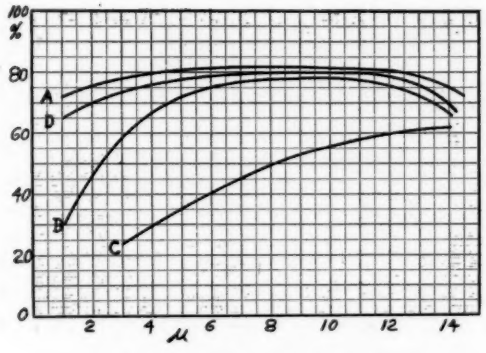

FIG. 4. Transmission of silver chloride.

A. Sheet .020" thick
B. Sheet .020" thick AgS coated
C. Sheet .012" thick multiple crystal ingot
D. Sheet .50" thick

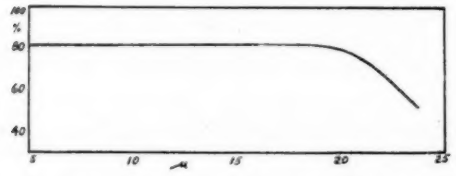

FIG. 4a. Transmission limits of silver chloride in the infra-red. Sample 4.75 mm thick.

Silver chloride sheet up to .5 inch in thickness shows no appreciable loss in transmission value.

It is to be noted however that the transmission behavior of single crystal sheet is different from that of multiple crystal sheet. Single crystal sheet shows full transmission value from 1 micron on, whereas the sheet prepared by Fugassi and McKinney starts at a low value and increases somewhat in the longer wave-length region. The transmission data up to 14 microns has been verified by a number of workers.[7]

The region beyond 14 microns has recently been measured by Earle K. Plyler, in charge of infra-red measurements of the National Bureau of Standards. With his kind permission this data is being included. It is to be noted that transmission continues up to full value to 20 microns. Figure 4a gives the transmission curve as determined on a sample 4.75 mm thick.

[7] Van Zandt Williams, American Cyanamid Company, J. J. Shipman, B. F. Goodrich Company, F. P. Hochgesang, Socony-Vacuum Laboratories.

PROTECTION OF SILVER CHLORIDE SHEET

Since the discoloring of silver chloride is caused by ultraviolet light, various coatings can be applied to the finished sheet or optical part to cut off ultraviolet, or even most of the visible light without seriously interfering with infra-red transmission.

SILVER SULPHIDE COATING

Silver sulphide coating is applied to the sheet or film by dipping in a dilute solution of alkali sulphide before the last pass through the rolls. A hard glossy coating results which is resistant to abrasion and effectively offers protection comparable to the inert character of silver sulphide. The intensity of these coatings can be varied from a light amber color to a coating heavy enough to all but extinguish a bright filament of a 100-watt clear tungsten lamp. (See Fig. 5.)

SELENIUM COATING

This method of protecting silver chloride was suggested by Pfund.[8] The selenium is sputtered on by the usual high vacuum method used for coating optical materials.

Gilsonite coatings afford excellent protection and finds application in cases where it is not practical to coat a finished optical part with silver sulphide.

RESISTANCE OF COATED FILMS

Various tests were devised to determine the resistance of coated films to the elements usually encountered in their use. Solarization tests were carried out under a 15-watt G. E. sterilizing

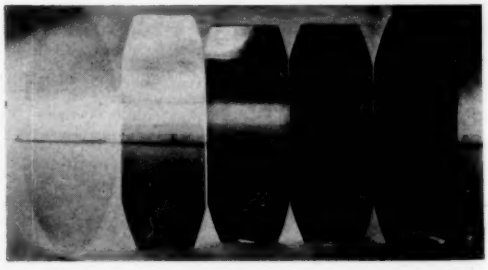

FIG. 5. Silver chloride sheets coated with Ag_2S.

[8] A. H. Pfund, J. Opt. Soc. Am. **23**, 375 (1933).

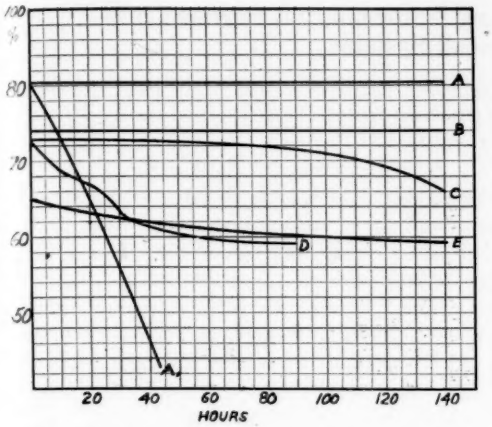

FIG. 6. Radiometric measurements (100° source) of infra-red transmission of silver chloride exposed to fluorescent lamp (2500A).

A. Uncoated—Unexposed
A1. Uncoated—Exposed
B. AgS coated—Unexposed
C. AgS coated—Exposed
D. Selenium coated. (Exposed to direct sunlight)
E. AgS coated and gilsonite

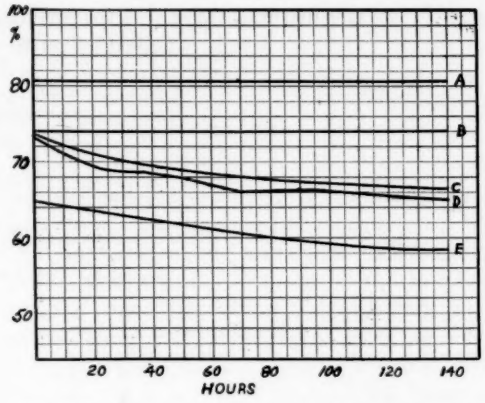

FIG. 7. Radiometric measurements (100° source) of infra-red transmission of silver chloride, exposed alternately to fluorescent lamp (2500A) and 10 percent salt solution.

A. Uncoated—Unexposed
B. AgS coated—Unexposed
C. AgS coated—Exposed
D. Selenium coated—Exposed
E. AgS coated and gilsonite—Exposed

mercury vapor lamp. The principal ultraviolet ray of this lamp is 2500A mercury line. Distance from the lamp was six inches. Saline immersion tests were carried out in 10 percent salt solutions. Two series of tests were run. Figure 6 gives the series of radiometric measurements (100° source) of transmissions at intervals during the exposure of the samples. Figure 7 gives a similar series of measurements on samples exposed by alternate exposures to ultraviolet light and salt water immersions. The alternate exposures being approximately of twelve hours duration.

It will be noted that the sulphide coated and selenium coated film both are reasonably well protected.

USES OF SILVER CHLORIDE

The unusual character of silver chloride together with its infra-red transmission makes this a very valuable optical material for the infra-red field. Most of the usual optical shapes suitable for infra-red can be fabricated by impact pressing with chrome plated dies. Dies prepared with prevision optical surfaces will produce finished optical parts which require no further polishing. Lenses and prisms for infra-red work made by impact pressing should make the problem of replacement parts relatively inexpensive.

Fuoss[6] has described an absorption cell using silver chloride windows. The material should lend itself well to the construction of a variety of cell shapes. It should even be possible to shape a complete cell to close tolerances by one pressing operation. The insoluble character of silver chloride to many solutions as compared with rocksalt and potassium bromide should be an advantage.

ACKNOWLEDGMENT

The author wishes to acknowledge the valuable assistance given on this problem by various members of the Harshaw Organization.

Many time consuming and seemingly impossible tasks have been very materially lightened by the able assistance of Robert E. Price.

A Differential Method for Introducing Non-Spherical Surfaces into Calculation of Optical Systems

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(Received October 28, 1946)

DURING recent years the designer of optical instruments constantly has been facing the problem of further improving the efficiency of optical systems. He has been elaborating photographic lenses with high aperture ratios and wide angular fields, wide-angle lenses with increased aperture ratios, eyepieces with increased fields, etc.

Solving problems of this kind usually leads to complex systems containing many components. This makes the computer, while looking for other ways, investigate the rationality of introducing non-spherical refracting surfaces into optical systems. The equation of such a surface may contain any number of independent coefficients. Suitable choice of them is required in order to reduce the number of spherical surfaces necessary for correcting the aberrations of a system. Problems relating to the *practice* of ray tracing through non-spherical surfaces have been treated in different papers and books. Long ago there was also solved the problem of determining the third-order aberration coefficients for systems of non-spherical surfaces.[1,2] Problems concerning *the method* of introducing non-spherical surfaces into the calculation of optical systems have been paid much less attention.

Some processes of calculation, offered by certain authors, relate, mainly, to the correction of aberrations either of the principal rays of oblique pencils for *several* field angles (for instance, the correction of distortion), or of aberrations of a beam with a finite aperture but for a single inclination. In the latter case the authors aim to correct, chiefly, the spherical aberration and sometimes the sine condition.

1. STATEMENT OF THE PROBLEM

The introduction of non-spherical surfaces for a simultaneous improvement of the correction of *several* aberrations and, in particular, those of axial and thick oblique pencils, complicates the problem of computing essentially, and demands the elaboration of a special method. This is understood as the elaboration of criteria allowing the solution of the following problems in the early stage of calculation without having recourse to rather complicated and laborious ray tracing through non-spherical surfaces:

1. To find out from the analysis of the structure of axial as well as oblique pencils which spherical surface of the system is to be shaped into a non-spherical one.

2. To find out from the analysis mentioned above the shape of a non-spherical surface without having recourse to any supplementary ray tracing through non-spherical surfaces.

3. To evaluate, though approximately, the influence of the introduced non-spherical surface upon the remaining aberrations of the system.

The intersection of a non-spherical surface and a meridional plane forms a curve the equation of which let be represented by an algebraic series

$$f(x)=y^2=a_1x+a_2x^2+a_3x^3+\cdots+a_kx^k. \quad (1)$$

Let the origin of a system of Cartesian coordinates be placed in the vertex of the curve, the x axis coinciding with axis of the optical system. The coefficients a_1 and a_2 may be expressed as follows:

$$\begin{aligned} a_1&=2r, \\ a_2&=-(1+b), \end{aligned} \quad (2)$$

where r represents the radius of curvature at the vertex of the curve; b is the "deformation coefficient" introduced by Schwarzschild[1]; in the case of conic sections $b=-e^2$.

The coefficients a_1 and a_2 represent parameters for the correction of aberrations in Seidel's region; in particular, by the right choice of them correction of the first-order aberration coefficients can be obtained.

The choice of the function $f(x)$ is entirely arbitrary. It is only important that the expansion

[1] Schwarzschild, *Untersuchungen zur geom. Optik, I* (Berlin, 1905).
[2] M. Rohr, *Die Bilderzeugung in Optischen Instr.* (Berlin, 1904).

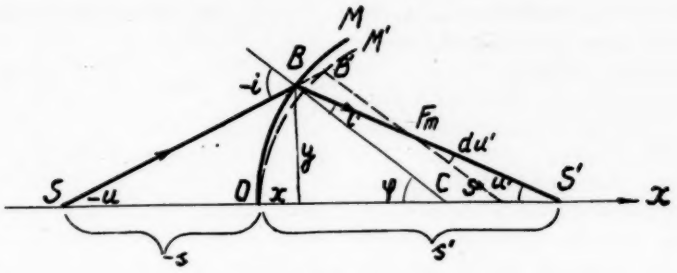

FIG. 1.

OM—a meridional section of the non-spherical refracting surface.
OM'—a meridional section of the spherical refracting surface.
SB—a ray incident on a non-spherical surface.
BS'—the refracted ray.

of the function in a series of ascending powers of x contains terms of odd and even powers with respect to x.

When the origin of coordinates coincides with the vertex of the non-spherical surface, the free term of the expansion must be absent. With regard to other properties, the function $f(x)$ may belong to any class of trigonometrical, exponential or other functions. Certainly, it would be of interest to find out a function $f(x)$ the operating of which leads to a simplification of the calculating process or possessing other essential advantages.

Let a ray *SB*, lying in the meridional plane (Fig. 1) encounter a non-spherical surface.

The coordinates of the point B of intersection of the curve *OBM* and the ray *SB* may be derived from Eq. (1), and the equation of the ray *SB* is

$$y=(s-x)\tan u. \tag{3}$$

The symbols s and u are explained in Fig. 1; s' and u' represent the corresponding coordinates of the ray after its refraction at the surface *OBM*.

Let this ray after traveling through the remaining part of the system (not shown in Fig. 1) reach some plane in *the image space*, say, the Gaussian plane, at a point the ordinate of which be l'.

The features of the proposed method are as follows.

By taking a system of spherical or non-spherical surfaces of the second-order corrected for the third-order aberrations we fixed the first two coefficients a_1 and a_2 in series (1) (see Eq. (2)). Determining variations of the ordinate $l'(dl')$ in the image space in terms of variations of the coefficients $a_3, a_4 \cdots$ of the non-spherical surface, we fix a direct relation between the changes of the coordinates and consequently between the aberrations of the ray and the changes of the coefficients in series (1) (insofar as the ray path in the paraxial region remains unaltered).

Thus finding the partial derivatives dl'/da_k for several rays (of an axial as well as of an oblique pencil) to be corrected, and analyzing the found magnitudes of the derivatives dl'/da_k we may solve the inverse problem: assuming beforehand any desired changes dl' of the coordinates of different rays in the image space to compute the corresponding magnitudes of the coefficients $a_k(k=3, 4\cdots)$ in Eq. (1).

It will be clear from the following that the proposed method immediately secures the possibility of obtaining answers to the three principal questions that have been stated at the beginning of the paragraph. This can be accomplished with comparatively little calculating work.

Getting answers to the same three questions by applying the usual method of systematical trials to the introducing of every new coefficient a_k in series (1) would necessitate immediate tracing rays through non-spherical surfaces. Consequently, solving equations of high orders would be necessary to find out the coordinates of the refracted ray. This is usually done by one of the methods of successive approximations, iterations, etc.

All the more work is necessary when applying

the latter method that the coefficients a_3, $a_4 \cdots$ influence differently upon higher order aberrations of axial and oblique pencils.

As a result it often happens that after a good deal of calculation one finds that there is, in general, no favorable solution.

In conclusion it may be noted that the application of a differential method to the solution of the problem of finding out the influence of each refracting surface of an optical system upon its aberrations is quite natural. In 1904 v. Rohr[2] investigated the problem of finding the influence of slight alterations of the radii of curvature of spherical surfaces upon spherical aberration, the sine condition and upon the chromatic variation of the focal point of a system in the region of third-order aberrations. Further research in this direction was accomplished by M. Russinov and C. Rabinovitch. They also dealt with a system of spherical surfaces, but the derivatives of the ray coordinates with respect to the parameters of the system they determined for aberrations of beams of any aperture and inclination, i.e., beyond Seidel's region.

2. THE CONDITION OF STABILITY OF THE "EXIT COORDINATES" OF A RAY REFRACTED AT A NON-SPHERICAL SURFACE

Let any three coefficients in series (1), say a_m, a_n, and a_p, vary by da_m, da_n, and da_p. Evidently, the two conditions of unalterableness of the exit coordinates u' and s' of a ray refracted at a non-spherical surface are

$$\left.\begin{aligned} d[f(x)] &= 0, \\ d\left[\frac{\partial y}{\partial x}\right] &= d\left[\frac{1}{\tan\varphi}\right] = 0 \end{aligned}\right\}. \tag{4}$$

As it has been taken that $dx = 0$, the first of these conditions, being applied to series (1), is

$$d[f(x)] = x^m da_m + x^n da_n + x_p da_p = 0. \tag{5}$$

The second condition may be represented in the form

$$d\left[\frac{\partial f(x)}{\partial x}\right] = mx^{m-1}da_m + nx^{n-1}da_n + px^{p-1}da_p = 0. \tag{6}$$

Solving (5), (6) conjointly, we get

$$\left.\begin{aligned} da_p &= -\frac{n-m}{p-m}x_0^{n-p}da_n, \\ da_m &= -\frac{p-n}{p-m}x_0^{n-m}da_n, \end{aligned}\right\} \tag{7}$$

where x_0 is a fixed coordinate of the point at which the ray encounters the surface.

It follows from (7) that insofar as $da_n = 0$,

$$da_m = da_p = 0.$$

Thus, by variation of no less than three coefficients of the non-spherical surface, the exit coordinates of a ray refracted at the surface may be kept unalterable.

Variations of the three adjacent coefficients a_{k-1}, a_k, a_{k+1} in series (1) being found, Eq. (7) gives us

$$\left.\begin{aligned} da_{k+1} &= -\frac{1}{2x_0}da_k, \\ da_{k-1} &= -\tfrac{1}{2}x_0 da_k. \end{aligned}\right\} \tag{8}$$

While finding variations of the three coefficients in series (1), one of them may be taken arbitrarily as the two Eqs. (7) or (8) bind three differentials.

The correlations (7) or (8) indicate the existence of a parameter which we shall name the "special parameter." The designer of optical systems can make use of it in some cases, for solving certain problems of correction, analyzing the specific structure of the beam in the image space.

In particular, the application of the "special parameter" may be useful when computing wide-angle systems. Here, keeping the Petzval coefficient and Seidel's third sum magnitude constant does not yet secure the "stability" of the correction of a thin sagittal astigmatic pencil during variations of the coefficients in series (1). This is true even in the case when the meridional beam with finite aperture remains well corrected. In fact, the radius of curvature r_s of a non-spherical surface determining the location of the sagittal focus is equal to the length of the normal BC (Fig. 1) to the curve at the point of incidence of the ray.

It can be seen from the drawing that

$$\langle BC\rangle_{kl}=r_s=\frac{y}{\sin\varphi}. \tag{9}$$

Thus the conditions (8) being satisfied, the angle φ and the coordinate y prove stable during variation of the coefficients in series (1), and consequently the magnitude r_s remains unaltered too.

3. CORRELATION BETWEEN THE PARTIAL DERIVATIVES OF THE ABERRATIONS OF A RAY IN THE IMAGE SPACE WITH RESPECT TO THE COEFFICIENTS OF THE NONSPHERICAL SURFACE

Let two partial derivatives of the aberrations of a ray in the image space be determined by one of the possible ways stated later

$$D_m=\frac{\partial l'}{\partial a_m} \quad \text{and} \quad D_n=\frac{\partial l'}{\partial a_n}, \tag{10}$$

where dl' is the variation of the ordinate l' of the intersection point of a ray in the image space and some plane, say, the focal plane of the optical system; ∂a_m, and ∂a_n are variations of the coefficients a_m, a_n in series (1).

Let us find a dependence allowing to determine the remaining partial derivatives D_p, D_r, D_s, etc. without having recourse to any complicated supplementary ray tracing. Here

$$D_p=\frac{\partial l'}{\partial a_p}, \quad D_r=\frac{\partial l'}{\partial a_r}, \quad D_s=\frac{\partial l'}{\partial a_s}, \text{ etc.} \tag{11}$$

∂a_p, ∂a_r, ∂a_s, etc. denote the variations of the pth, rth, sth coefficients in series (1).

Finding variations of the coefficients of the meridional curve (1) only, one may regard the variations dl' of the ordinate l' in the image space as functions of the variations da_m, da_n, da_p, etc. This functional dependence, evidently, is

$$dl'=D_m da_m+D_n da_n+D_p da_p+\cdots. \tag{12}$$

In the preceeding paragraph it has been shown that for keeping the path of a given ray constant (the coordinates being x_0, y_0) during variation of the coefficients in series (1) it is necessary and sufficient that the variations of no less than *three* coefficients in series (1) be found. These variations are to satisfy the limitations (7).

Taking advantage of the conditions (7) and assuming in expression (12) $dl'=0$ we find

$$\frac{\partial l'}{\partial a_p}=\frac{n-p}{n-m}x_0^{p-m}\frac{\partial l'}{\partial a_m}+\frac{p-m}{n-m}x_0^{p-n}\frac{\partial l'}{\partial a_n}. \tag{13}$$

The variations of any three adjacent coefficients in series (1) being found, say, of the coefficients with indices $k-1$, k and $k+1$, one may determine from (13), assuming $m=k-1$, $n=k$, $p=k+1$,

$$\frac{\partial l'}{\partial a_{k+1}}=2x_0\frac{\partial l'}{\partial a_k}-x_0^2\frac{\partial l'}{\partial a_{k-1}}. \tag{14}$$

Thus, if, by any way, any two partial derivatives are determined, for instance, $\partial l'/\partial a_{k-1}$ and $\partial l'/\partial a_k$, one may find from (14) the derivative $\partial l'/\partial a_{k+1}$, etc.

4. AN ANALYTICAL STATEMENT OF THE DIFFERENTIAL METHOD OF CALCULATION

The essential of the proposed method of calculation has in general been stated in Section 1. Here is an analytical statement of the method.

Let an optical system consisting of spherical or any other surfaces of the second order, be investigated to find out the rationality of introducing one or several non-spherical surfaces of higher orders. Under the latter are understood surfaces of which the equations of the meridional sections contain terms from series (1) of higher than the second powers with respect to x.

Let the non-spherical surface be introduced in order to change the exit coordinates of certain rays in the image space, say, rays No. 1, 2, 3, etc. These rays may belong to any pencils of different inclinations.

Let us denote dl_1', dl_2', dl_3', etc. the changes of the coordinates l_1', l_2', $l_3'\cdots$ of the rays No. 1, 2, $3\cdots$ in the image space. Let $D_1^{(1)}$, $D_2^{(1)}$, $D_3^{(1)}$, etc. be denoted the partial derivatives of the coordinates l_1' with respect to the coefficients a_1, a_2, $a_3\cdots$, relating to ray No. 1; the same for ray No. 2 which is denoted $D_1^{(2)}$, $D_2^{(2)}$, $D_3^{(2)}$, etc.

We come thus to a system of linear equations

$$\begin{aligned}
dl_1' &= D_1^{(1)}da_1+D_2^{(1)}da_2+D_3^{(1)}da_3+\cdots,\\
dl_2' &= D_1^{(2)}da_1+D_2^{(2)}da_2+D_3^{(2)}da_3+\cdots,\\
dl_3' &= D_1^{(3)}da_1+D_2^{(3)}da_2+D_3^{(3)}da_3+\cdots,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
dl_N' &= D_1^{(N)}da_1+D_2^{(N)}da_2+D_3^{(N)}da_3+\cdots.
\end{aligned} \tag{15}$$

Taking into consideration that the variations dl_1', dl_2', $dl_3' \cdots dl_n'$ are usually to be obtained, but approximately it cannot be regarded rational to bring down the problem of finding the variations da_1, da_2, $da_3 \cdots$ to solving a system of linear equations of the type (15).

It is more rational, evaluating the magnitudes of the partial derivatives D_1', D_2', $D_3' \cdots$, to make use of the variations but of the coefficients in series (1), representing the "most active" parameters when solving the problem of correction in every given case. Here it is being made the more exact choice of magnitudes of variations of the coefficients da_1, da_2, $da_3 \cdots$ at which, with more or less approximation, the necessary variations dl_1', dl_2', $dl_3' \cdots$ in the system of Eqs. (15) are obtained.

In case the "introducing of non-sphericity" into a given surface does not lead to the result wanted, the investigation is proceeded with the aim of finding out the rationality of "introducing non-sphericity" into some other spherical surface of the system.

Frequently, while making use of preliminary data relating to the correction of a system of a given type, analyzing the ray paths of different pencils through the system of a given type, analyzing the components of Seidel's coefficients at certain surfaces of the system, the designer derives some necessary hints for solving the above-mentioned problem. In this way he may find out which of the spherical surfaces of the system causes aberrations of higher orders, and, consequently, which of the surfaces is to be investigated in the first turn for introducing non-sphericity.

When several non-spherical surfaces are introduced into a system, there comes another system of equations instead of (15)

$$\left.\begin{aligned} dl_1' &= \textstyle\sum_1^k D_k^{(1)} da_k + \sum_1^n E_n^{(1)} db_n + \cdots, \\ dl_2' &= \textstyle\sum_1^k D_k^{(2)} da_k + \sum_1^n E_n^{(2)} db_n + \cdots, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ dl_N' &= \textstyle\sum_1^k D_k^{(N)} da_k + \sum_1^n E_n^{(N)} db_n + \cdots, \end{aligned}\right\} \quad (16)$$

where D, E, etc. and their corresponding indices denote the partial derivatives of the coordinates of the rays in the image space determined accordingly with respect to coefficients a, b, etc. of the following series

$$\left.\begin{aligned} y^2 &= a_1 x + a_2 x^2 + a_3 x^3 + \cdots, \\ y^2 &= b_1 x + b_2 x^2 + b_3 x^3 + \cdots. \end{aligned}\right\} \quad (17)$$

Any of these series represents the equation of a meridional curve belonging to some non-spherical surface.

In the proposed method comparatively the most laborious is the determination of the magnitudes of the two "initial" partial derivatives D_k and D_{k-1} of the ray coordinates in the image space with respect to any two coefficients in series (1). The partial derivatives with respect to the remaining coefficients in the series are then calculated with the aid of a simple recurrent formula (14)

$$D_{k+1} = 2x_0 D_k - x_0^2 D_{k-1}. \quad (18)$$

A statement of the method of determining the two "initial" partial derivatives D_{k-1}, D_k entering the formula (18) follows.

5. METHODS OF DETERMINATION OF THE TWO "INITIAL" PARTIAL DERIVATIVES D_{k-1} AND D_k

Different methods for determination of the "initial" partial derivatives $D_{k-1} = \partial l'/\partial a_{k-1}$ and $D_k = \partial l'/\partial a_k$ may be invented; the seemingly most practical of them will be stated here, i.e., methods demanding the least calculating work. They may be recommended for use in different cases according to the specific features of the problem solved.

1. A method of immediate determination of the partial derivatives D_{k-1} and D_k in the case when the non-spherical surface is represented by the last surface of the optical system.

2. A method based upon the application of the laws of optics of collinear relationship to thin meridional pencils.

3. A simplified method of determining the partial derivatives D_{k-1} and D_k in case the introduced deformations of the surface be small.

4. A method of immediate determination of the partial derivatives D_{k-1} and D_k, based upon the application of trigonometrical tables describing how changes of the parameters of a system influence its aberrations, the tables being composed for systems of spherical surfaces.

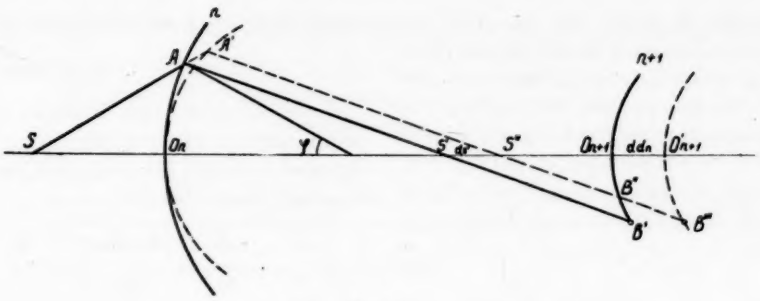

FIG. 2.

OnA'—a meridional section of the spherical refracting surface.
OnA—a meridional section of the non-spherical surface.
$SA'B'''$—the ray path after refraction at the nth spherical surface OnA'; the ray meets the $n+1$ spherical surface at a point B'''.
SAB'—the ray after refraction at the nth non-spherical surface OnA; at a point B' the ray meets the $n+1$ spherical surface parallelly displaced over the distance dd_n.
The points B' and B''' are equidistant from the optical axis.

In this paper only the last of the above-mentioned methods is treated.

6. A METHOD OF IMMEDIATE DETERMINATION OF THE PARTIAL DERIVATIVES D_{k-1}, D_k, BASED UPON DATA FROM TRIGONOMETRICAL TABLES DESCRIBING HOW CHANGES OF THE PARAMETERS OF A SYSTEM INFLUENCE ITS ABERRATIONS

For the system of spherical surfaces under investigation, let a table be composed by ray tracing, describing how changes of the components of the system (say, radii and thicknesses) influence its aberrations. It may often happen that no special computation of such a table be necessary. Use can be made of the data derived by the designer while correcting the system.

Let some spherical surface belonging to the system (say, the nth) be investigated in order to find out the rationality of its deformation, i.e., of introducing retardations from sphericity.

Let $a_{n,k}$ denote the kth coefficient in series (1) relating to the nth surface of the system. It will be proved that the magnitude of the partial derivative $D_{n,k}=dl'/da_{n,k}$ may be determined when the magnitudes of certain two partial derivatives of the ray coordinate l' with respect to two certain parameters of the system are known. The analysis shows that it is, probably, most convenient to choose as parameters the radius r_n of the surface under investigation and the distance between the vertexes of the nth and $n+1$ surfaces.

Thus, let the magnitudes of two partial derivatives be

$$\partial l'/\partial r_n \text{ and } \partial l'/\partial d_n. \tag{19}$$

Considering the variation dl' as a function of the variations dr_n, da_{nk}, and dd_n, we have

$$dl'=\frac{\partial l'}{\partial r_n}dr_n+\frac{\partial l'}{\partial a_{n,k}}da_{n,k}+\frac{\partial l'}{\partial d_n}dd_n. \tag{20}$$

The magnitudes of the variations dr_n, $da_{n,k}$ and dd_n may be chosen in such a way that some ray $A'B''$ (Fig. 2) after being refracted at the surface O_nA' under deformation travels through the remaining part of the optical system (i.e., from the $n+1$th surface and on) in the same path as before deforming the nth surface (the surface O_nA). Evidently, here at the magnitude dl' of the left-hand part of Eq. (20) will prove equal to zero:

$$\frac{\partial l'}{\partial r_n}dr_n+\frac{\partial l'}{\partial a_{n,k}}da_{n,k}+\frac{\partial l'}{\partial d_n}dd_n=0. \tag{21}$$

This condition may be fulfilled by satisfying the following two demands:

(a) The ray $A'S''$ after being refracted at the deformed surface O_nA' should remain parallel to the ray AS' refracted at the spherical surface O_nA:

$$\varphi_n=\text{const.} \tag{22}$$

(b) The height at which the ray $A'S''$ encounters the $n+1$th surface should remain equal to the height at which the ray AS' encounters the same surface. For this purpose it is necessary to change the distance d_n between the vertexes of the nth and the $n+1$th surfaces ($d_n = O_nO_{n+1}$) by dd_n (Fig. 2). This will render equal the ordinates of the points B''' and B'; here at, evidently,

$$dd_n = ds_n'. \tag{23}$$

Taking advantage of these two conditions one may express the differentials dr_n and dd_n through the differential $da_{n,k}$ and making use of (21) derive the relation sought for

$$\frac{\partial l'}{\partial a_{n,k}} = \frac{(k+E)x_n^{k-1}}{2(1+E)} \cdot \frac{\partial l'}{\partial r_n} - \frac{(k-1)x_n^k \sin\varphi_n \sin(u_n' - u_n)}{2(1+E)y_n \cos i_n \sin u_n'} \cdot \frac{\partial l'}{\partial d_n} \tag{24}$$

where

$$E = -\frac{x_n \cos u_n}{2y_n \cos i_n}[2\tan u_n \cos\varphi_n + \sin\varphi_n f''(x)], \tag{25}$$

thus making use of the trigonometrical table describing the influence of finite differences Δd_n and Δr_n upon the changes of the ray coordinate $\Delta l'$ in the image space and taking advantage of the approximate equalities

$$\partial l'/\partial r_n \simeq \Delta l'/\Delta r_n \quad \text{and} \quad \partial l'/\partial d_n \simeq \Delta l'/\Delta d_n. \tag{26}$$

The magnitude of $\partial l'/\partial a_{n,k}$ may be determined from (24). Now, if necessary, the remaining derivatives $\partial l'/\partial a_{n,k+1}$, $\partial l'/\partial a_{n,k+2}$ etc., may be determined. The simplest way to do it is to make use of the general relation (18).

In case of deforming a spherical surface for which $f''(x) = -2$ we find from (25)

$$E = E_{\text{sph}} = -(x/y)\tan i. \tag{27}$$

It can be shown that in many cases, at any preliminary calculations, one may take instead of the exact formula (24), neglecting the second item in its right-hand part

$$\frac{\partial l'}{\partial a_{n,k}} \simeq \frac{(k+E)x_n^{k-1}}{2(1+E)} \cdot \frac{\partial l'}{\partial r_n}. \tag{28}$$

The formula (28) is an approximation obtained as a result of neglecting the influence of dx, dy upon variations of the coordinates ds' and du' of the refracted ray and, considering the influence upon the latter, by rotating the normal of variations of the angle dy.

7. CONCLUSIONS

The stated method of calculation secures the possibility:

(1) To get answers to the principal questions, formulated by the author in Section 1, with comparatively little calculating. Methods applied usually to investigation of changes in the structure of ray pencils, arising from the introduction of every following coefficient a_k of series (1), demand immediate ray tracing through non-spherical surfaces of high orders and, consequently, solving equations of high powers.

(2) To evaluate immediately the influence of coefficients of higher orders in series (1) upon the structure of the pencil when the influence of any two coefficients and, in particular, of the first two of them, upon the structure of the pencil is known.

(3) To make use of the data of usual ray tracing through a system of *spherical* surfaces.

The latter may be considered as a particular advantage of the method.

Making Fresnel Off-Axis Parabolic Mirrors by the Evaporation Technique

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A technique was worked out for making Fresnel off-axis parabolic mirrors by selective deposition of evaporated aluminum onto spherical mirrors. Tests were made to determine the extent and the nature of the interference between the light reflected from the different zones. When the illuminating slit was parallel to the principal radius there was no undesirable interference and no appreciable reduction in resolving power.

INTRODUCTION

STRONG[1,2] has described a procedure for figuring mirrors by controlled deposition of evaporated aluminum; he has also pointed out that a limitation of this method is set by the appearance of bloom on aluminum deposits of thicknesses more than about two microns. The present work was undertaken to discover methods of reducing the thickness required. It was found that a possible way is to form a Fresnel mirror. The experimental details developed for making off-axis parabolic mirrors by this method will be discussed and an evaluation given of the mirrors in terms of their performance in several experimental arrangements, special attention being given to the effects of interference and to resolving power.

PRINCIPLES OF THE METHOD

If a circle given by

$$X^2-2XR+Y^2=0 \qquad (1)$$

intersects the parabola,

$$X=Y^2/2R' \qquad (2)$$

at the points $(0, 0)$ and $(X_0, \pm Y_0)$ with $Y_0 \ll R$, the separation of these two curves is given approximately by,

$$\Delta X=Y^2(Y_0^2-Y^2)/8R^3 \qquad (3)$$

and the maximum separation at Y equal to $0.707\,Y_0$ is

$$\Delta X_m=Y_0^4/32R^3. \qquad (4)$$

In conventional polishing methods a spherical mirror is parabolized by removal of material, but from Eq. (3) it is evident that the same result can be accomplished by deposition of material to the thickness ΔX. This has been done successfully by evaporation of aluminum onto spherical blanks.[2] The method fails, however, when ΔX_m exceeds about two microns because evaporated aluminum of greater thickness takes up a peculiar crystal formation called "bloom" which greatly diminishes its reflectivity. This limitation may be avoided by forming a Fresnel mirror. In the experimental work the immediate object was to produce off-axis mirrors, therefore all the diagrams will be shown with that type of mirror. Figure 1A illustrates how a sphere is

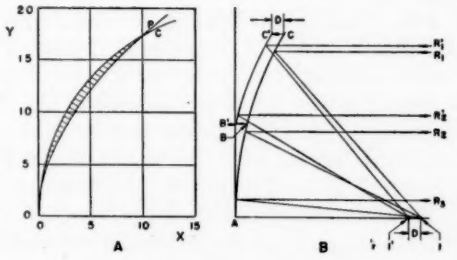

FIG. 1. In A is shown a section of a circle C with a radius equal to 20 units and a parabola P given by $X=Y^2/30$. The two curves intersect at X equal to 0 and to 10 units. The shaded area represents the deposit required to "parabolize" the circle. B describes the "Fresnel mirror" idea: The rays R_1, R_2, and R_3 incident on the continuous parabolic section ABC are united after reflection to form an image at I. $B'C'$ is formed by displacing BC a distance D parallel to the axis. The rays R'_1 and R'_2 then form an image at I'. For an actual case D will be of the order of 15,000A and I and I' are essentially one image. Accordingly, a mirror with the broken figure $ABB'C'$ will give the same performance (except for possible interference effects) as one with the continuous figure ABC. Following the terminology used in designating cellular lenses, a mirror with a broken first-surface will be referred to as a "Fresnel mirror."

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[1] John Strong, *Procedures in Experimental Physics* (Prentice-Hall, Inc., New York, 1945).
[2] John Strong, J. Opt. Soc. Am. **26**, 153 (1936).

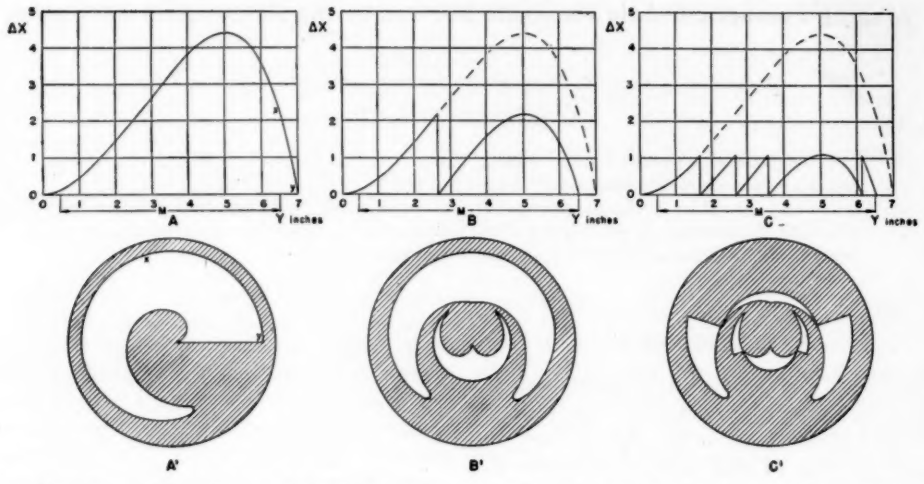

FIG. 2. Three types of deposition and the diaphragm used to obtain each. Graph A shows the full deposit required by Eq. (3); the associated diaphragm, A', is straight-forward in design. Graph B shows a deposit with one step dividing the mirror surface into two zones; the diaphragm B' is so designed that only half as much aluminum must be evaporated for B as for A. Graph C shows a deposit of five zones. With the diaphragm C' an amount of aluminum equal to that required for B must be evaporated. The diaphragm openings might be changed to meet the taste of the experimenter, it being only required that $\theta(Y)$ have the proper value.

parabolized by a deposit ΔX and B explains the origin of the term "Fresnel mirror."

Consideration of Fig. 2 will explain the procedure: In A is shown a graphical representation of Eq. (3) with Y_0 equal to 7 inches; below the X axis is shown the position of a six-inch spherical mirror which will become an off-axis parabolic after receiving the aluminum deposit ΔX. By the argument given in Fig. 1B, it follows that the deposit shown by the solid curve in Fig. 2B will produce a mirror giving the same geometrical performance as that with the deposit of Fig. 2A. In Fig. 2C is shown the deposit for a larger number of zones. It is evident that this procedure reduces the thickness of the aluminum required to parabolize the original spherical mirror.

The experimental arrangement was similar to that used by Strong, but with better geometry. Aluminum was evaporated from a helical tungsten filament bent into the form of a circle ten inches in diameter. This filament was of 40-mil wire and contained 30 loops of one-fourth inch diameter which was just sufficient to prevent the molten metal from running together and collecting at one point. Each loop was loaded with two one-half inch pieces of 50-mil aluminum bent U shaped so as to hang on the lower part of the loop. To prevent excessive heating in the evaporation chamber the filament was divided into three equal parts each of which could be heated independently. These parts were held on rods mounted through insulators on the base plate.

The mirror blanks were supported on a rotor which was concentric with, and parallel to the plane of the filament, the rotor being turned by a small electric motor contained in the vacuum system. The six-inch spherically figured Pyrex blanks were placed one-half inch off-axis. Directly below the mirror surface was the diaphragm. The relative position of these parts is shown in Fig. 3. Most of the evaporations were made with the filament to mirror distance equal to six inches.

DESIGN OF THE DIAPHRAGMS

The angular opening of the diaphragms can be calculated directly: Let Y equal the distance from the axis, then if $F(Y)$ describes the filament radiation characteristics, and if $M(Y)$ is the desired deposit, the angular opening is given by

$$\theta(Y) = M(Y)/F(Y). \tag{5}$$

$F(Y)$ for a circular filament has been worked out

and is given in the form of a table for various parameters;[1] $M(Y)$ is given by Eq. (3).

In Fig. 2A′ is shown a form conveniently used to obtain the deposit of Fig. 2A. It was found advantageous to cut the diaphragm for a larger Y_0 than required so as to reduce the length of the defining edge. Thus, the length x to y on the diaphragm lies beyond the remote edge of the mirror and accordingly x to y on the graph is not significant. The diaphragms for the deposits of Fig. 2B and C are shown below the respective graphs.

If a broad source of aluminum is used when making Fresnel mirrors it is necessary to have the defining edge of the diaphragm very close to the mirror surface to minimize the lack of sharpness in the shadow edge of the circular arc in the diaphragm. For the arrangement used in these experiments Fig. 3 shows that the width of the defective areas will be approximately equal to the mirror to diaphragm separation. In Fig. 4A is shown the theoretical contour of the zone edge and also the actual contour for a separation "D" of the diaphragm and mirror. Figure 4B shows a constructional design that allows a great reduction in D: A diaphragm with an opening larger than required was cut from flat one-eighth inch aluminum to support the defining edge which was of 10-mil aluminum. Round head screws with their heads filed down to a few thousandths thickness were used to hold the two together. It was found that when two different metals were used the differential expansion during evaporation was often enough to warp the unit and cause it to scratch the mirror surface. Several attempts were made to make the diaphragm fit the mirror contour as in Fig. 4C but the gain did not seem worth the extra trouble. A better approach might be to reduce the size of the aluminum source.

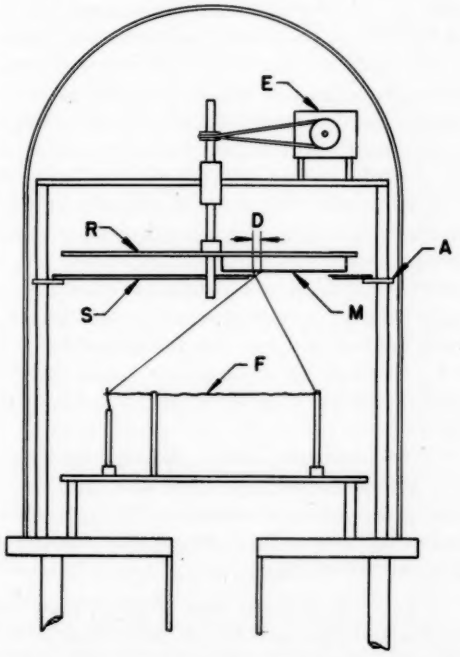

FIG. 3. The experimental arrangement. A six-inch spherical mirror, M, is shown supported on the under side of the rotor, R. A small electric motor, E, turns R at about 40 r.p.m. The circular filament, F, is suspended on insulated vertical rods. Parts A are used to place the diaphram, S, immediately below the mirror surface. It is evident that the defective arc forming the boundary between zones will have a width D about equal to the diaphram and mirror separation.

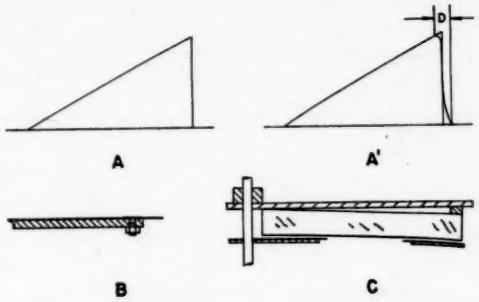

FIG. 4. In A is shown the theoretical contour of an aluminum deposit; A′ shows the same theoretical contour together with the actual contour which gives a defective region of width D. In B is shown a design for reducing the mirror and diaphragm separation. The arrangement of C attempts to make the diaphragm fit the spherical mirror blank.

PERFORMANCE AND RESULTS

From geometrical considerations mirrors with the different figures A, B, and C of Fig. 2 should all give the same performance. Because of the order of the step thickness, however, some difficulty might be expected because of interference of light reflected from the different zones. This expectation was carefully checked.

Test methods for off-axis mirrors have been given earlier.[3] In Fig. 5 are shown possible rela-

[3] L. G. Schulz, J. Opt. Soc. Am. **36**, 588, (1946).

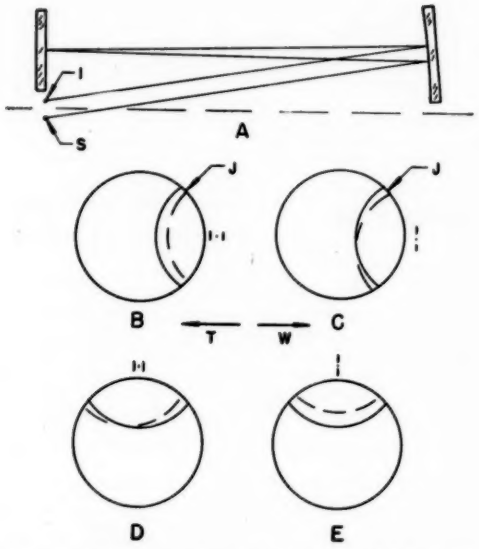

FIG. 5. The arrangements for testing off-axis mirrors with an optical flat. In A is shown the relative position of the mirror and flat, while B to E show the slit position as seen along the axis. The solid arc on each surface represents a zone boundary; the broken line is the image of this boundary. The displacement due to parallax is about equal to the separation of the illuminating slit and its image. In any of the diagrams the slit and its image may be interchanged but the boundary image displacement will always be toward the slit image side of the principal radius. Parallax should be kept small to cause the zone boundary and its image to overlap as much as possible. The arrows below B and C indicate the two possible directions in which the knife-edge may cut the image: T is motion toward the mirror and W is away from the mirror.

tive orientations of the optical parts when testing with an optical flat. In A is shown the position of the mirror and the flat, while B to E show in detail the slit and image position as seen along the axis. B and C show the slit and its image perpendicular to the principal radius.* B, the usual arrangement, is not good for testing purposes because of parallax effects. C on the other hand is the best arrangement for testing because the parallax is perpendicular to the principal radius and therefore along the arc defects. D and E show the slit and its image parallel to the principal radius, D being the more convenient arrangement and the best for using off-axis mirrors. Tests were also made without an optical flat including trials with the slit and its image

* The principal radius extends from the axis to the remote edge of the mirror; it passes through the center of a circular mirror.

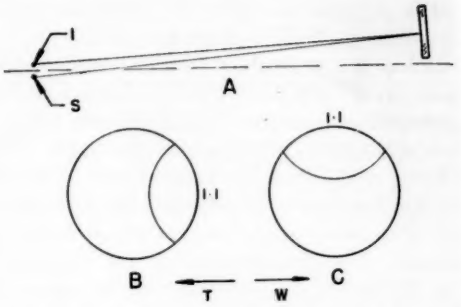

FIG. 6. The arrangement for testing without an optical flat. There will be no parallax effects. T and W have the same meaning as in Fig. 5, but here only apply to B.

parallel as well as perpendicular to the principal radius, as in Fig. 6.

The slit source used during the test was about 0.04 mm wide and 1.0 mm long; illumination was from a six-volt tungsten filament lamp. The position of the knife edge used to examine the image of this slit was read with micrometer screws accurate to 0.005 mm. When using an optical flat the knife edge was set to give the most even darkening of the mirror and then the sources of color pattern were looked for on the mirror surface. When testing without flat the knife edge was set to give last darkening on the boundary of the two zones, and then any color pattern was observed.

Spherical mirrors six inches in diameter and with radii of curvature equal to 80″ and 100″ were used as blanks. The maximum thickness, ΔX_m, for mirrors of these two sizes using the three types of deposit of Fig. 2 are given in Table I.

Interference effects were detected by observation of any color changes from the natural radiation of incandescent tungsten. The principal effects observed while testing these mirrors can be summarized thus:

1. When the step size was 19,000A there were no interference colors of any kind in any of the arrangements of Figs. 5 and 6.
2. None of the mirrors regardless of figure or step size showed interference effects when tested in arrangements D and E of Fig. 5 or C of Fig. 6.
3. For Fig. 5 arrangements B and C and Fig. 6 B there was an effect related to the direction in which the image was cut by the knife edge.

When the motion was toward the mirror (indicated by the direction arrow *T*) no colors could be observed for Fig. 5 B and C. For Fig. 6 B there was occasionally a just detectable coloring at the ends of boundry between two zones.

4. Interference phenomena could be observed for step thicknesses of 10,000A and 5000A in arrangements B and C of Fig. 5 and B of Fig. 6, but only when the knife edge motion was in the *W* direction, away from the mirror. When using a flat, the interference was practically all confined to the region of overlapping zones marked *C*. Without a flat, colors showed to about one-half inch on each side of the zone boundary where last darkening occurred.

The conditions giving no observable interference are easily understood: First, if the step heights exceed about three wave-lengths no color will arise for the same reason that a thin transparent film of this thickness or greater shows no interference fringes. Second, if the slit is parallel to the principal radius, interference bands are roughly perpendicular to the slit length and spread in the direction of the slit edges. The effect is the same as using a very wide slit in arrangements B or C—no color could be seen.

Other interesting interference phenomena were observed with these mirrors but details cannot be given here. Resolving power was also investigated. A convenient arrangement for making some of these optical studies was to mount a slit of about 0.03 mm width on the knife edge motion and then pass the slit through the plane of the image. In addition to the usual diffraction shown by colored fringes there were also dark and light band patterns, in particular for small zone widths. In general, as the overlapping of zones increased, the interference effects multiplied.

The spherical mirror blanks used were of the commerical quality designated as "good to one-eighth wave-length." The diaphragms used were accurate enough to keep the defects introduced by parabolization well below the one-eighth wave-length magnitude even when ΔX_m amounted to 18,000A. In fact, when the mirrors were examined in arrangements Fig. 5D it was difficult even to detect parabolization defects, whereas the original errors in the spherical blanks could in general then be easily measured. If perfect spherical blanks are used slit widths as small as 0.01 mm might be used with profit.

EVALUATION

Some advantages of using the arrangement of Fig. 5D for off-axis mirrors have been pointed out previously.[3] In general, this arrangement with the slit parallel to the principal radius greatly reduces the image broadening due to arc defects in the mirror figure. The tests just described show that Fresnel mirrors used in this arrangement give no undesirable interference phenomena, and in addition there is no appreciable loss of resolving power because the mirror aperture perpendicular to the slit length has not been reduced by the Fresnel zones. The Fresnel method of parabolization therefore offers a means of avoiding the limitation set by bloom formation in thick deposits of aluminum. Also this method makes it unnecessary to cut extremely accurate diaphragms since the absolute thickness of aluminum, and likewise the absolute error, is kept small. Even a rough diaphragm will give excellent parabolization if the deposit is thin enough.

If, however, the Fresnel mirror must be used in arrangement Fig. 4B (with the slit perpendicular to the principal radius) it has serious disadvantages. Most important, the resolving power is reduced by approximately one over the number of surface elements, thereby destroying the advantage of the wide aperture mirror. Further, although the interference is extremely small relative to the total light reflected, it makes the mirror of questionable value for infra-red spectroscopy where the wave-length might greatly exceed the step size. It may be possible to eliminate interference effects by wide masking over the zone boundaries but this would seriously reduce the total effective area; nothing can be done to avoid the reduction in resolving power other than using arrangement Fig. 5D.

The tests here described were not the most general ones possible. Illumination was confined

TABLE I.

Radius of curvature	ΔX_m for the three types of deposit in Fig. 2 (In Angstrom units)		
	A	*B*	*C*
100″	19,000	9500	4750
80″	37,000	18,000	9250

to slit sources and the main interest was with mirrors to be used with a flat. The two arrangements with the slit parallel and perpendicular to the principal radius represent extremes; conceivably the mirror might be used with some value other than 0° and 90° for this angle. A more complete answer would be given by examining a Fresnel on-axis mirror. As already explained[3] the difference between the extremes is increased as the axis is moved out further beyond the mirror periphery.

The limits of the Fresnel method were not determined. By placing the aluminum source on the axis it would be possible totally to eliminate the boundary defect between the zones. When making mirrors of small "*f*-number" and large aperture, the width of the zone would become increasingly small. Whether or not any new difficulties will arise with very small zone widths cannot be completely predicted. For many off-axis mirrors made by polishing methods the resolving power is limited by figure defects. The evaporation technique makes it possible to produce mirrors of such quality that the theoretical limit is approached.

CONCLUSIONS

The experiments have shown that Fresnel off-axis parabolic mirrors made by the evaporation technique when used with a slit source parallel to the principal radius have no undesirable interference defects and a resolving power about equal to that of a mirror of the same aperture with an unbroken figure. This is true for all wavelengths and for any step thickness of the mirror zones. In addition this method makes it easily possible to make the figure so nearly perfect that the theoretical resolving power is approximated. The evaporation technique, therefore, should be a helpful method in making mirrors more cheaply and of better quality to meet the increased demand.

If it is necessary to use the mirror with the slit perpendicular to the principal radius there will be a reduction in resolving power and under some circumstances an undesirable interference effect. Before using a Fresnel mirror under such conditions the experimenter must consider the quality of the image desired. It should be kept in mind that usually figure defects of off-axis mirrors made by polishing methods determine the usable minimum slit width, and that as a consequence a Fresnel mirror of good quality may give higher resolution. Also the amount of light in the interference is extremely small relative to the total reflected from the mirror, careful tests being necessary even to find it, and then only for steps of small height. Accordingly, for many purposes Fresnel mirrors may be used without concern given to the orientation of the slit source.

The experimental work was done at the University of Michigan and at the Evaporated Metal Films Corporation at Ithaca, New York. The author wishes to thank Professor Robley C. Williams for suggestions and Mr. J. C. Ufford for permission to publish these results.

Ultraviolet Phosphors and Fluorescent Sun Tan Lamps

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An investigation was undertaken of phosphors capable of converting primary ultraviolet radiation (Hg line, 2537 angstroms) to radiation in the erythemal or sun tan ultraviolet region (2900–3200 angstroms). Several types of alkaline earth silicates and phosphates activated with lead or thallium were found to be practical. Among these was a calcium phosphate activated with thallium and containing about 1.3 moles of calcium per mole of phosphate. This phosphor not only showed a good initial erythemal emission, but when used in an experimental fluorescent lamp maintained a high percentage of this emission for the several hundred hours tested. The lamp was constructed of special glass and coated with the above phosphor, but otherwise was similar to commercial fluorescent lamps. In addition, the investigation showed some similar types of phosphors to be efficient in the neighboring "black-light" ultraviolet region (approximately 3400–3800 angstroms), and to give more emission in that region than the cerium-activated phosphors commonly used at present. Some discussion of sun tan lamp and fixture design is also included.

A. INTRODUCTION

IN order to fill the need for a low cost, high efficiency sun tan lamp, an extensive investigation of ultraviolet emission by means of fluorescence was undertaken several years ago by the Sylvania Electric Products Inc. of Salem, Massachusetts.

The commercially available sun lamps at present are usually high intensity mercury discharge lamps consuming 100 to 500 watts or more of power and having a very low efficiency of conversion of this wattage into available sun tan radiation. The results of this investigation indicate that a fluorescent type of sun tan lamp is practical, and that it can be made to have an efficiency of conversion considerably higher than that of the older style lamps, so that the same sun tan radiation can be produced by approximately one-half of the wattage. An added advantage of a fluorescent sun tan lamp would be that no specialized equipment would be necessary for its operation. The lamp could be substituted into any present fluorescent lamp fixture.

The problem of fluorescent conversion is mainly a physical-chemical problem of fluorescent powders or phosphors and their manufacture. On the phosphor depends the efficiency of conversion of energy to the wave-length and brightness desired. When a phosphor is subjected to a radiation of a given wave-length, the fluorescent radiation emitted is always of a longer wave-length than the incident radiation. A common example of this is the irradiation of certain phosphors by black-light, having a wave-length of about 3400–3800 angstroms, and the spontaneous emission by these phosphors of visible light, of wave-lengths anywhere from 4000 (blue) to 7000 (red) angstroms. Another is the irradiation of certain other phosphors by ultraviolet light having a principal radiation of 2537 angstroms as produced inside an ordinary fluorescent lamp by a low pressure mercury discharge, and the spon-

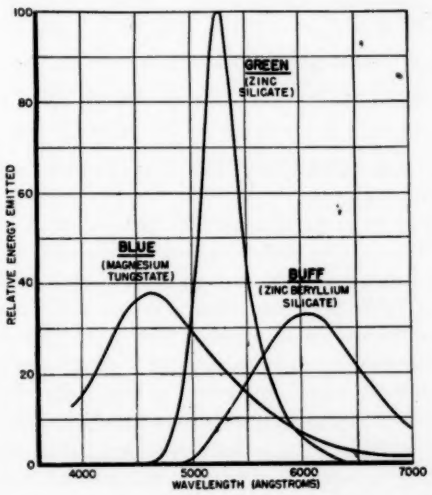

FIG. 1. Typical visible emissions of common phosphors used in fluorescent lamps.

* Now at U. S. Naval Research Laboratory, Anacostia, Maryland.

TABLE I.

Phosphor type		Relative total ultraviolet emission 2800–4000A	Peak wave-length
Matrix	Activator		
KCl	Tl	4.0 ma	3000A
$MgSiO_3$	Tl	1.0	—
$CaSiO_3$	Tl	1.3	3700
$AlSiO_3$	Tl	1.4	3800
$ZnSiO_3$	Tl	0.8	3800
$BeSiO_3$	Tl	2.4	3850
$CdSiO_3$	Tl	0.8	—
$BaSiO_3$	Tl	3.6	3200
$MgSiO_3$	Tl, Pb	1.0	—
$CaSiO_3$	Tl, Pb	4.6	3350
$ZnSiO_3$	Tl, Pb	1.0	3800
$CdSiO_3$	Tl, Pb	0.8	—
$SrSiO_3$	Tl, Pb	1.3	—
$BaSiO_3$	Tl, Pb	7.1	3200
$Ba \cdot Sr \cdot Ca \cdot SiO_3$	Tl, Pb	3.6	—
$BaSiO_3$	Tl, Pb, Sb	6.9	3200
$BaSiO_3$	Pb	7.2 (e_1)	3200
$BaSi_2O_5$	Pb	15.5 (e_2)	3500
$CaSiO_3$	Pb	6.0	—
$CaFSiO_3$	Tl	6.8	3350
$BaFSiO_3$	Tl	3.3	3400
$BaFSiO_3$	Pb	18.0 (d)	3500
$CaFSiO_3$	Tl, Pb	10.3	3400
$BaFSiO_3$	Tl, Pb	16.0	3500
$BaCaFSiO_3$	Tl, Pb	2.9	—
$ZnAl_2O_4$	Tl	0.9	—
$ZnAl_2O_4$	Tl, Pb	0.6	—
$CaAl_2O_4$	Tl, Pb	1.1	—
$Ca_3(PO_4)_2$	Tl	10.4 (c)	3300
$CaHPO_4$	Tl	9.9	3300
$Ca_3(PO_4)_2 \cdot KCl$	Tl	3.9 (a)	3000
$CaKPO_4$	Tl	6.9 (b)	3150
$Ca(K_2PO_4)_2$	Tl	1.0	3800
$Ba_3(PO_4)_2$	Tl	0.6	—
$Ba \cdot Ca \cdot PO_4$	Tl	7.5	3250
$Ca_3(PO_4)_2$	Tl, Pb	3.9	—
$Ca_3(PO_4)_2$	Pb	1.9	—
$Ca_3(PO_4)_2$	Ce	16.2	3600

taneous emission by these phosphors of visible light as above.

The type of phosphor chosen is entirely dependent on its desired use. Considerations other than wave-length and brightness of emission must also be taken into account, such as the effect of air, ozone, gaseous discharges, heat, etc., either during manufacture or during operation within the lamp. Important as well is its adaptability to mechanical treatment, such as grinding and milling into paints or other suspensions, (including control of particle size and uniformity of the applied coating).

Typical curves of common visible-light producing phosphors are shown in Fig. 1, where the brightness of emission is plotted against wave-length. These were all irradiated with a low pressure mercury arc discharge of 2537 angstroms, as is usual within ordinary fluorescent lamps.

B. ULTRAVIOLET FLUORESCENCE

Two general ranges of ultraviolet wave-lengths are of commercial interest: The black-light region of 3400 to 3800 angstroms, and the erythemal (sun tan) region of 2900 to 3200 angstroms. Phosphors capable of converting 2537 primary radiation to black-light radiation can be of a calcium phosphate type activated with cerium. These, however, are of rather unstable structure and subject to deterioration during the processing of a fluorescent lamp. In the course of the present investigation on ultraviolet producing phosphors, a new type of barium silicate phosphor was developed which was proven much more efficient and stable for black-light emission. This will be identified more fully along with others investigated in the data which follows.

The investigation of phosphors capable of converting 2537 primary radiation to radiation in the erythemal band was, however, the primary object of this work. As a basis of reference for brightness and spectral distribution measurements, a phosphor consisting of potassium chloride activated with thallium chloride was made up according to the method reported by Pringsheim.* This phosphor showed a spectral emission spread from 2750 up to 4500 angstroms when activated by 2537 angstrom radiation, with a peak brightness between 2800 and 3200, centering on 3000 angstroms.

This material was, however, extremely hygroscopic, and in the absence of special protective coatings was found to be unadapted for fluorescent lamp use, as it produced rapid deterioration within the lamp. This resulted in a black coating on the bulb after only a few hours burning. Since, however, the emission spectrum was in the desired erythemal range, this type of phosphor was chosen as a temporary reference standard.

As a means of measuring experimental phosphors to determine the approximate ultraviolet light output prior to making them up into

* Péter Pringsheim, Rev. Mod. Phys. **14**, 132 (July, 1942).

fluorescent lamps, the following plaque method was standardized upon:

a. Total ultraviolet brightness was measured by pressing a portion of the powder into a three-inch diameter cup or plaque to present a uniform surface, irradiating this surface with 2537 angstroms from a ring-shaped quartz low pressure mercury vapor lamp placed directly above the plaque, and picking up the emitted fluorescence from the plaque by means of a quartz-window photo-cell. Over this photo-cell window was placed one or more Corning light filters to allow only emitted wave-lengths of a chosen ultraviolet band to reach the photo-cell and be recorded on the microammeter. Corning filters Nos. 9863, 5970, 9700 and 774 were used in various combinations as desired.

b. The spectral energy distribution of the light emitted by the powders on these plaques was measured by replacing the photo-cell in the above assembly with a spectrograph, and making a photographic record in the form of a spectrogram. From this a densitometer curve was made to show the energy distribution with wave-length.

Investigations were made on a large number of types of phosphors to determine roughly the possibilities of various combinations of matrix materials and activators. In Table I, no attempt has been made to indicate the exact proportions of the ingredients as used, but, rather, to show the general type of phosphors investigated, and the optimum results obtained from each series of investigations. Chemical formulas have been used only for the sake of brevity and are not intended to imply exact molal proportions.

It should be noted that these do not necessarily represent similarly weighted results on all these types of phosphors, since more development work was done on those which initially looked promising than was done on those which were initially poorer.

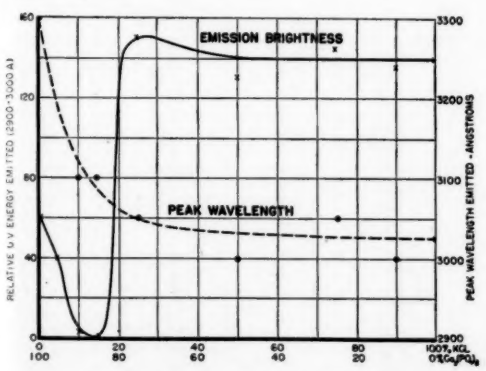

FIG. 2. $Ca_3(PO_4)_2 \cdot KCl$ (Tl) series activated with thallium. Change in sun tan emission with increasing chlorides.

In this tabulation the total ultraviolet (2800–4000A) emission was measured by the photo-cell method described above using Corning filters Nos. 9863 and 9700 in combination. The peak wave-length was determined by densitometer curves taken from the spectrograms.

It is interesting to note that the addition of manganese as a second activator to some of the above phosphors, for example, to lead-activated calcium silicate, as done by John B. Merrill and William E. Stewart of Sylvania, produces a double peak in emission, one in the ultraviolet and another in the visible.

C. MANUFACTURE OF ULTRAVIOLET PHOSPHORS

Of these groups investigated, five were chosen as basic types for further development and testing:

(*a*) $Ca_3(PO_4)_2 \cdot KCl$ (Tl)	for its 3000A peak wave-length, despite its low total ultraviolet brightness.
(*b*) $CaKPO_4$ (Tl)	for its increased total ultraviolet brightness over (a).
(*c*) $Ca_3(PO_4)_2$ (Tl)	for its combination of fairly good brightness and reasonably low wave-length.
(*d*) $BaFSiO_3$ (Pb)	for its extremely high total ultraviolet brightness.
(*e*) $BaSiO_3$ (Pb) series	for its high total ultraviolet brightness at more than one wave-length.

(The KCl (Tl) and the $Ca_3(PO_4)_2$ (Ce) were used as reference standards only, the one for erythemal and the other for black-light regions.)

These five phosphors are listed below according

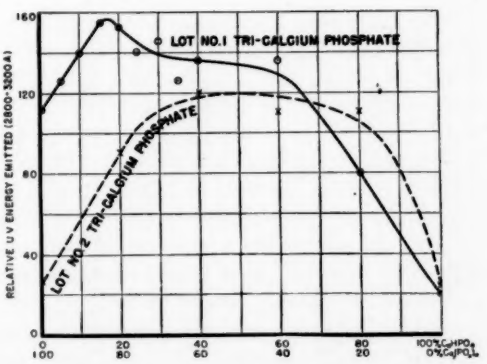
LOT NO. 1 TRI-CALCIUM PHOSPHATE
LOT NO. 2 TRI-CALCIUM PHOSPHATE
RELATIVE U V ENERGY EMITTED (2800-3000 A)
100% Ca3(PO4)2
0% Ca3(PO4)2

FIG. 3. $Ca_3(PO_4)_2$ series activated with thallium. Change in sun tan emission with increasing calcium-to-phosphorus rates.

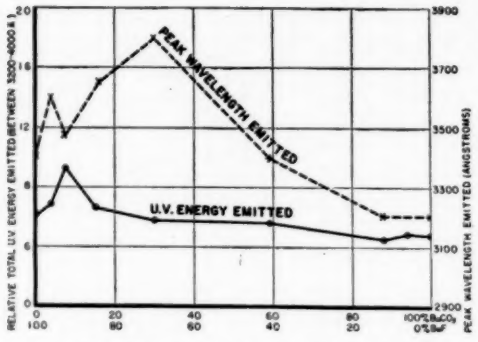

FIG. 4. $BaFSiO_3$ series activated with lead. Change in wave-length and total ultraviolet energy emitted with increasing carbonates.

to their optimum formulas as subsequently determined, all ingredients being C.P. chemicals.

Phosphor type	*Ingredients*	*Processing*
(*a*) $Ca_3(PO_4)_2KCl$ (Tl)	460 g $Ca_3(PO_4)_2$ 460 g KCl 46 g Tl_2SO_4	Milled 3 hr. with 1750 cc acetone, dried, and fired at 1090°C for 1 hour.
(*b*) $CaKPO_4$ (Tl)	93.7 g K_2CO_3 256.3 g $CaHPO_4$	Base material prefired 18 hr. at 900°C, then added:
	115 g $Ca_3(PO_4)_2$ 11.5 g Tl_2SO_4	Final fired at 900°C for 1 hr. after milling 3 hr. in acetone.
(*c*) $Ca_3(PO_4)_2$ (Tl)	675 g $Ca_3(PO_4)_2$ 34 g Tl_2SO_4	Milled 3 hr. with 1900 cc acetone, dried, and fired at 950°C for 17 hr.
(*d*) $BaFSiO_3$	140 g BaF_2 12 g $BaCO_3$ 82 g SiO_2 3 g PbO_2	Milled 3 hr. with acetone, dried, and fired at 920°C for 7 hr.
(e_1) $BaSiO_3$ (Pb)	540 g $BaCO_3$ 210 g H_2SiO_3 10.2 g PbO_2	Milled 3 hr. with acetone, dried, and fired at 1100°C for 4 hr.
(e_2) $BaSi_2O_5$ (Pb)	372 g $BaCO_3$ 300 g H_2SiO_3 15 g PbO_2	Milled 8 hr. with acetone, dried, and fired at 1100°C for 16 hr.

Considerable variations in these formulas were investigated in the course of their development:

(*a*) *$Ca_3(PO_4)_2 \cdot KCl$ (Tl).*—The variation in the ratio of $Ca_3(PO_4)_2$ to KCl in the basic formula given above shows both a shift in the peak wavelength and a marked change in the total fluorescent energy emitted in the ultraviolet region. The combination of these two factors results in an extreme change in the fluorescent energy emitted in the erythemal band of 2900–3000 angstroms. The accompanying curves of Fig. 2 show these effects.

(*b*) *$CaKPO_4$ (Tl).*—Optimum brightness and minimum wave-length (3150 angstroms) both occurred at the basic formula given above.

TABLE II.

BaF_2 C.P.	BaF_2 special	$BaCO_3$ C.P.	H_2SiO_3 C.P.	SiO_2 fired	PbO_2	Tl_2SO_4	Relative total ultraviolet brightness	Peak emission wave-length
—	—	170	62	—	1.5	6	4.7	3200A
10	—	162	62	—	1.5	6	4.8	3200
22	—	150	62	—	1.5	6	4.5	3200
68	—	96	62	—	1.5	6	5.4	3400
111	—	48	62	—	1.5	6	5.6	3800
132	—	24	62	—	1.5	6	6.5	3850
140	—	12	62	—	1.5	6	9.0	3500
148	—	6	62	—	1.5	6	6.8	3550
155	—	—	62	—	1.5	6	5.8	3400
155	—	—	62	—	—	6	3.0	3400
140	—	12	62	—	1.5	6	9.0	3500
140	—	12	95	—	1.5	6	9.4	3500
—	140	12	62	—	1.5	6	12.4	3500
—	140	12	95	—	1.5	6	14.3	3500
—	140	12	62	—	1.5	—	10.4	3500
—	140	12	—	53	1.5	6	14.3	3500
—	140	12	—	53	3.0	—	14.2	3500
—	140	12	—	82	3.0	—	17.0	3500

(*c*) $Ca_3(PO_4)_2$ (*Tl*).—Variations in the ratio of calcium to phosphorus were shown to produce a marked effect on the brightness of the ultraviolet emission of this type of phosphor. The proportions of calcium and phosphorus were varied by blending tri-calcium-phosphate and secondary-calcium-phosphate (as purchased), and as shown by the accompanying curves of Fig. 3, there is a very definite optimum formulation. The apparent optimum formula will vary with individual lots and sources of secondary and tertiary calcium phosphates, but there is evidence to show that this variation is dependent only on the actual compositions of these phosphates as originally precipitated. Analysis shows the optimum point to be at a ratio of about 1.3 moles of calcium to 1.0 mole of phosphate. Either analysis of these ingredients or experimental blending of them in varying proportions (as represented by the curves in Fig. 3) can be used as a control procedure for setting up the proper formulation for a commercial phosphor. The presence of iron as an impurity materially lowers the peak brightness of this type phosphor: in Fig. 3, for example, lot No. 1 was found to contain .001 percent Fe and lot No 2 0.003 percent Fe. Thallium sulfate is preferable to other thallium salts as an activator.

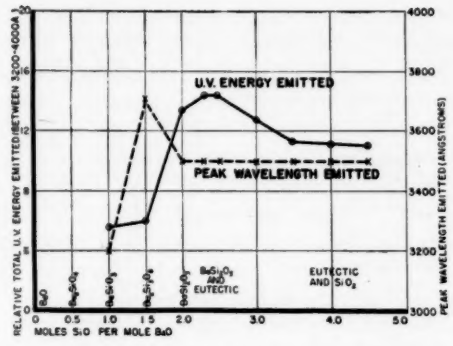

FIG. 5. $BaSiO_3$ series activated with lead. Change in wave-length and total ultraviolet energy emitted as a function of crystal structure.

(*d*) *BaFSiO*$_3$ (*Pb*).—This type of phosphor is extremely sensitive to variations in processing. The presence of a small amount of $BaCO_3$ in the formula was found to be of prime importance in producing a bright phosphor up to certain limits, but an excess of $BaCO_3$ was found to be detrimental. A shift in the emitted wave-length also accompanied the addition of $BaCO_3$ as shown in the curves of Fig. 4. No BaF on the market was found suitable for this process, and it was necessary to prepare this material specially in this laboratory by the reaction of pure $BaCO_3$, HF, and NH_4OH before the optimum-brightness phosphor could be obtained. Prefiring of the silicic acid to silica before compounding was found to produce a 15 percent increase in brightness of the phosphor, while increasing the amount of silica used from a mole ratio of 1:1 to a mole ratio of 1.5:1 (SiO_2:BaO), still further increased the brightness by 20 percent without affecting the emitted wave-length. Lead is a primary activator for this type of phosphor, whereas thallium shows only a small degree of activation (with or without lead). These variations are shown in detail in the accompanying Table II. The time-temperature schedule in firing this type of phosphor is critical, since it is easily subject to loss in brightness by overfiring. As an example of

TABLE III.

Formula: 50 g H_3SiO_3 2.4 g PbO_2 plus:	Mole ratio BaO:SiO_2	Relative total ultraviolet brightness	Peak emission wave-length
149 g $BaCO_3$	1:1.0	5.7	3200A
99 $BaCO_3$	1:1.5	6.2	3750
74 $BaCO_3$	1:2.0	13.5	3500
71 $BaCO_3$	1:2.1	13.9	3500
65 $BaCO_3$	1:2.3	14.5	3500
59 $BaCO_3$	1:2.5	14.5	3500
55 $BaCO_3$	1:2.7	13.9	3500
51 $BaCO_3$	1:2.9	13.2	3500
49 $BaCO_3$	1:3.0	12.8	3500
43 $BaCO_3$	1:3.5	11.4	3500
37 $BaCO_3$	1:4.0	11.1	3500
33 $BaCO_3$	1:4.5	11.1	3500

TABLE IV.

Mole ratio BaO:SiO_2	Composition of solid at room temperature	Peak emission of phosphors
1:0.0	BaO crystals	—
1:0.5	Ba_2SiO_4 (ortho) crystals	—
1:0.9	Ba_2SiO_4-$BaSiO_3$ eutectic	—
1:1.0	$BaSiO_3$ (meta) crystals	3200A (dim)
1:1.4	$BaSiO_3$-$Ba_2Si_3O_8$ eutectic	—
1:1.5	$Ba_2Si_3O_8$ crystals	3750 (dim)
1:1.5–2.0	Mixed crystals	—
1:2.0	$BaSi_2O_5$ crystals	3500 (bright)
1:2.9	$BaSi_2O_5$-SiO_2 eutectic	3500 (bright)
0:*X*	SiO_2 crystals	—

TABLE V.

Phosphor group	Plaque readings on phosphor powder: Peak emission wave-length	Plaque readings on phosphor powder: Relative total ultraviolet energy 2800–4000 angstroms	Readings on finished lamps: Peak emission wave-length	Readings on finished lamps: Relative total ultraviolet energy 2800–4000 angstroms	Readings on finished lamps: Relative erythemal energy 2900–3000 angstroms	Readings on finished lamps: Relative black-light energy 3400–3800 angstroms
(*a*) $Ca_3(PO_4)_2 \cdot KCl$ (Tl)	3050	3.9	3050	11	700	
(*b*) $CaKPO_4$ (Tl)	3150	6.9	3200	17	800	
(*c*) $Ca_3(PO_4)_2$ (Tl)	3300	10.4	3350	30	800	
(*d*) $BaFSiO_3$ (Pb)	3500	18.0	3500			186
(e_1) $BaSiO_3$ (Pb)	3200	7.2	3200	17	400	
(e_2) $BaSi_2O_5$ (Pb)	3500	15.5	3500			184
(*f*) KCl (Tl)	for reference		3000	12	800	
(*g*) $Ca_3(PO_4)_2$ (Ce)	for reference		3600			133

this, the following time schedule is typical of this phosphor when fired at 920°C:

Time fired	*Relative brightness*
5 hr.	14.6
7 hr.	17.0
9 hr.	14.8
13 hr.	11.2

Chemical analysis of the fired powders indicated that no volatile SiF_4 was formed during firing, since all of the initial barium and silicon was accounted for, but that 77 percent of the initial fluorine was lost and replaced by oxide. Total loss in powder weight on firing was 6.22 percent.

(*e*) *$BaSiO_3$ (Pb)*.—This type of phosphor is subject to a change in wave-length of emission with a change in the ratio of BaO to SiO_2. It is not sensitive to changes in the amount of lead used as activator (within limits). Table III shows the results of these variables. These figures represent the average values of several tests run. Reference to the phase diagram of BaO-SiO_2* shows the stable compounds and eutectics, and indicates correlation between these crystal structures and the emitted wave-lengths (Table IV).

These correlations are shown in the curves of Fig. 5.

D. ADAPTABILITY OF THESE BASIC TYPES OF PHOSPHORS TO USE IN FLUORESCENT LAMPS

Representative lots of each of the selected types of phosphors, each manufactured according to its optimum formula as listed above, were prepared in lacquer suspension in the usual manner for coating fluorescent lamp bulbs. Optimum bulb coating densities were determined experimentally for each phosphor and the phosphors were then coated onto glass bulbs. Lime glass was used for the black-light phosphors and germicidal glass for the sun tan phosphors. This germicidal glass will pass all ultraviolet wave-lengths down to approximately 2000 angstroms. These bulbs were then made up into fluorescent lamps (having the usual construction of commercial fluorescent lamps except for the phosphor-coated bulbs) and were subjected to tests as follows.

Spectral energy distributions of the lighted lamps were measured, and densitometer curves were made of the spectrograms. The black-light lamps were tested for 3400–3800A emission, initially and during life, along with a standard type of black-light lamp for reference. The sun tan lamps were similarly tested initially for 2900–3000A emission, and also were tested for total ultraviolet energy (2800–4000A) initially and during life. Table V shows the results of the initial measurements made on these lamps, with reference also to the previous plaque measurements on the original phosphors, which show good correlation. The values within each column show true comparisons. The differences in methods of measuring the energy in the various wave-length bands preclude direct comparison between columns. Figure 6 shows how the ultraviolet fluorescent energy emitted by the black-light lamps is maintained during life. In comparative tests, the $BaSi_2O_5$ (Pb) phosphor lamps had higher emis-

* *International Critical Tables.*

sion after burning 2400 hours than either of the other types tested had after burning only 1000 hours. Figure 7 shows how the ultraviolet fluorescent energy emitted by the sun tan lamps is maintained during the first 1000 hours of life. The lamps made with the $Ca_3(PO_4)_2 \cdot KCl$ (Tl) type of phosphor showed complete failure of ultraviolet fluorescent emission during the first 24 hours of burning. The lamps made with the $CaKPO_4$ (Tl) type of phosphor showed about 40 percent maintenance of ultraviolet fluorescent emission after burning 1000 hours. Both of these groups of lamps turned quite black as they burned. The $BaSiO_3$ (Pb) type of phosphor showed an unusual trend in its total ultraviolet maintenance curve. Reference to Fig. 5 suggests the possibility of a shift in crystal structure during the burning of the lamp, although unfortunately no measurements of wave-lengths were taken during life to confirm this. The $Ca_3(PO_4)_2$ (Tl) type of phosphor showed reasonably high output values and good maintenance (82 percent at 1000 hours). All life test figures represent an average of a minimum of five lamps in each group.

E. LAMP AND FIXTURE DESIGN

The germicidal glass used for bulbs in the above sun tan lamp tests, which passes all wavelengths down to approximately 2000 angstroms, can be replaced for production work by other (soft) glasses, such as Corning No. 9821 which has a transmission cut-off substantially at 2800 angstroms. This latter type of glass has been proven satisfactory for manufacturing requirements. Germicidal glass, however, does transmit a larger percentage of the sun tan radiation, and if the phosphor coating is made thick enough on germicidal glass the radiations below the sun tan range (especially the 2537 mercury radiation) may be almost completely absorbed by the phosphor. Table VI shows in one special instance the increased absorption of these low wavelengths by heavier coatings of phosphor.

Erythemal tests were made on the writer's forearm to provide a practical check on the lamps. For these tests new 15-watt lamps were used which were similar to those made up for the life tests reported above, and a glass filter having a transmission cut-off at approximately 2700 angstroms was used to eliminate any effects whatsoever on the skin by a primary 2537 radiation from the mercury discharge.

For this particular test, a suitable mask was used to limit the exposed areas to narrow bands on the arm. Two 15-watt lamps of the group to be tested were mounted in a standard twin-socket floor model fluorescent lamp fixture, and the unmasked strip of the arm exposed for 30 minutes at a distance of $4\frac{1}{2}$ inches from the lamps. This exposure was found to be too intense for more than a small area of skin as it produced quite painful burns. The results, did, however, substantiate the differences between the various types of phosphors shown in the previous tests.

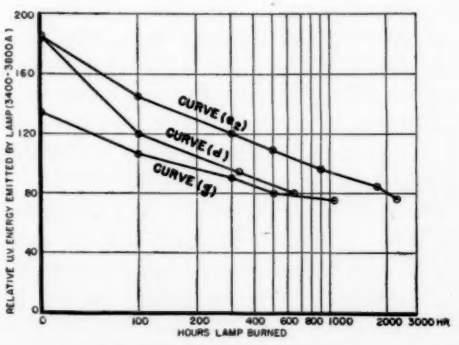

FIG. 6. Black-light lamps. Emission maintenance of three types of phosphors during life.

Curve (*d*) $BaFSiO_3$ (Pb) phosphor.
Curve (e_2) $BaSi_2O_5$ (Pb) phosphor.
Curve (*g*) $Ca_3(PO_4)_2$ (Ce) reference phosphor.

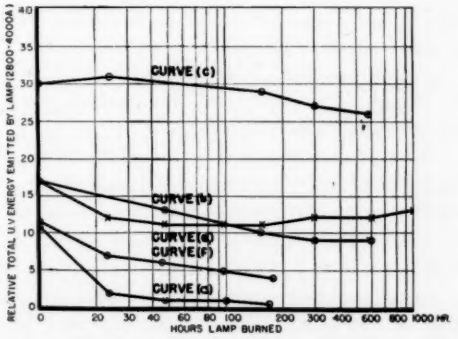

FIG. 7. Sun tan lamps. Emission maintenance of five types of phosphors during life.

Curve (*a*) $Ca_3(PO_4)_2 \cdot KCl$ (Tl) phosphor.
Curve (*b*) $CaKPO_4$ (Tl) phosphor.
Curve (*c*) $Ca_3(PO_4)_2$ (Tl) phosphor.
Curve (e_1) $BaSiO_3$ (Pb) phosphor.
Curve (*f*) KCl (Tl) reference phosphor.

TABLE VI.

$Ca_3(PO_4)_2$ (Tl) phosphor coating on bulb		Percent of total emission in range indicated		
		2000–2900A germicidal	2900–3200A erythemal	3200–4000A (by difference)
Heavy	(80.8)	3.7%	10.2%	86.1%
Light	(60.7)	13.8	9.7	76.5
No phosphor	(0)	87.0	3.7	9.3

TABLE VII.

Current used through lamp	Relative ultraviolet energy emitted (2800–3200A)
0.3 amp.	11
0.6	19
0.9	22
1.2	25

The $Ca_3(PO_4)_2$ (Tl) type of phosphor was used in the following.

Some twenty different persons were exposed to identical erythemal test conditions. These showed rather widely varying reactions. An exposure which on the most sensitive person produced an uncomfortable burn gave no perceptible erythema on the least sensitive person. It was, therefore, obvious that to produce a desired erythema, each person must determine his own exposure sensitivity. Consequently, any comparison of sun tan lamps must be made on the same person.

It was observed during the course of this investigation that care must be taken not only to protect the eyes from germicidal radiations which very quickly produce conjunctivitis, but to protect them also from prolonged exposures to erythemal radiation for the same reason.

The two-lamp fixture was next redesigned to obtain proper ultraviolet reflective properties by the use of aluminum reflectors, and to allow an increased amount of current to pass through the lamps by proper choice of electrical ballasts. At the same time the heater-cathode construction of the lamps was adjusted for operation at this higher current. By these means the effective erythemal output from the unit was greatly increased. The effect of the current increase alone is shown in Table VII.

Maintenance characteristics of the ultraviolet emission of these high current lamps during life show that, in the absence of forced cooling of the lamps in the fixtures, a current of 0.8 amp. through a so-called 15-watt size fluorescent lamp of special cathode construction is about optimum. This is 250 percent of the current used in the regular 15-watt fluorescent lamp, and lamps so made show an ultraviolet emission maintenance of about 80 percent after 1000 hours burning.

In a two-lamp fixture, with each lamp drawing 0.6 amp., the total wattage of the whole unit is 140 watts. The erythema produced by this unit on a given subject (exposure: 5 minutes at 18 inches) is fully as strong as that from a commercial mercury type lamp consuming 300 watts.

F. SPECIAL INSTALLATIONS AND APPLICATIONS

Installations of this type of sun tan lamp can be adapted to bathroom use along with a source of visible light, whereby an automatic 5-minute exposure is given each day during shaving, etc. In a wall installation in a shower, the waterproofing cover glass must be of an ultraviolet transmitting type, such as Corning No. 9700.

Commercial installations have recently been made in connection with the poultry business. Here the use of so-called germicidal glass bulbs is recommended, which allow the unconverted 2537A and lower wave-length radiations to pass through and act in a germicidal capacity. In fact, the ratio of germicidal to erythemal emission can be controlled by decreasing the thickness of the phosphor coat inside the bulb to convert less or more of the mercury discharge radiation (germicidal) into fluorescent radiation (erythemal).

Table VI shows a specific illustration of this. Lamps were made in germicidal glass bulbs and tested at a current of 0.7 amp. The coating density of the phosphor on the bulb was measured by the percent absorption of a ray of visible light passing through the coated bulb.

The authors are indebted to Mr. E. Curwen of the Sylvania Electric Products Inc. for spectrographic and life test measurements made during the course of this investigation; to Dr. J. H. Schulman of the same company for general data on visible-light phosphors; and to Mr. R. Hansen for engineering assistance in the laboratory.

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Use of a Monochromatic Colorimeter for the Study of Color Blindness

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A method and apparatus are described which make it possible to classify accurately defective color vision and to study the limens, with regard to shape, size, and orientation, of any observer. The instrument, which is described in detail, employs the principle of monochromatic colorimetry. The experimental arrangement is such that instrumental readings may be plotted directly upon the I.C.I. chromaticity diagram, so that the limens may be explored during the period of actual testing. Limens have been obtained for seven observers and for five colors distributed around the chromaticity diagram. The limens are characteristic of the observer, and those established for the several observers exhibit large and interesting differences. One observer was found to be a deuteranope, two were anomalous trichromats in significantly different degrees, and the remaining four observers having "normal" color vision were found to differ appreciably in their limens. The method, which is too slow for routine testing, would seem to be of value in fundamental studies of both normal and abnormal color vision, and in the testing of observers whose occupations involve possible loss of life or valuable property through incorrect color discrimination. Some practical implications are discussed.

INTRODUCTION

COLOR matching in the paper industry is usually rather exacting. The physical nature of paper is such that small differences in color are apt to be readily apparent to experienced observers having normal color vision, thus causing the color of paper to have importance comparable with that of far more costly commodities. The personnel involved in color problems include the "beater men" and others directly responsible for the control of color, salesmen, and quite often, research men and management. Disputes often arise among these men, even where it is known through testing and experience that the disputants have "normal" color vision. It has long been known that the commonly employed color blindness tests (such as the Ishihara and other pseudo-isochromatic tests, yarn tests, etc.) are not uniformly reliable in predicting the ability of persons correctly to see the color differences involved in the dyeing of paper. There appeared to be a need for research equipment which would furnish more complete and fundamental data on the extent and nature of the color differences that come into play when an observer attempts to match a given color by *approaches from a number of different directions*. In view of the natural limitations of colored bodies and of the difficulties encountered in preparing a suitable array of such bodies, the decision was made a few years ago to construct an apparatus in which a given color might be matched by mixing spectrum light with white light. Except for the absence of accurate photometric devices, the apparatus employed in the study was essentially a monochromatic colorimeter.

There is an abundant literature relating to color blindness. The essential facts of color blindness, the physical and psychological conditions for reliable testing, and the evolution of color blindness tests have recently been discussed by, respectively, Judd,[1] Dimmick,[2] and Murray.[3] In a series of four papers, Hardy, Rand, and Rittler[4–7] have published the results of extensive investigations of the Ishihara and Rabkin tests. MacAdam[8,9] has described a method and apparatus for determining visual sensitivities to color differences, and has presented data for normal observers on chromaticity limens at constant luminance. During the past few years the Color Aptitude Test Committee of the Inter-Society

* Now at the Farrand Optical Company, Inc., New York, New York.

[1] Deane B. Judd, J. Opt. Soc. Am. **33**, 294–307 (1943).
[2] Forrest Lee Dimmick, J. Opt. Soc. Am. **33**, 308–315 (1943).
[3] Elsie Murray, J. Opt. Soc. Am. **33**, 316–334 (1943).
[4] Le Grand Hardy, Gertrude Rand, and M. Catherine Rittler, J. Opt. Soc. Am. **35**, 268–275 (1945).
[5] Le Grand Hardy, Gertrude Rand, and M. Catherine Rittler, J. Opt. Soc. Am. **35**, 350–356 (1945).
[6] Le Grand Hardy, Gertrude Rand, and M. Catherine Rittler, J. Opt. Soc. Am. **35**, 481–491 (1945).
[7] Le Grand Hardy, Gertrude Rand, and M. Catherine Rittler, J. Opt. Soc. Am. **36**, 86–94 (1946).
[8] David L. MacAdam, J. Opt. Soc. Am. **32**, 247–274 (1942).
[9] David L. MacAdam, J. Opt. Soc. Am. **33**, 18–26 (1943).

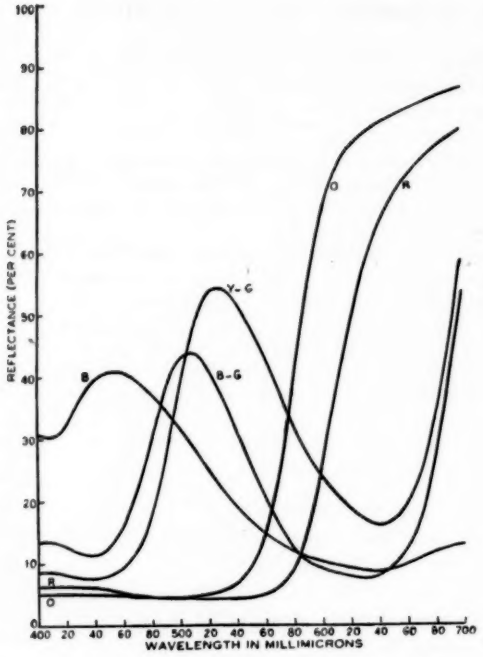

FIG. 1. Spectral reflectivity curves for colored papers used in this study.

Color Council has evolved an aptitude test for persons having normal color vision; a description of this test, together with statistical data, has recently been given by Dimmick.[10] The reader interested in a review of the field of colorimetry should consult the *Handbook of Colorimetry*[11] and the excellent report recently prepared by the Committee on Colorimetry of the Optical Society of America.[12]

It is not the purpose of the present work to evolve still another color blindness test. This article describes our method and equipment, and presents data obtained in an attempt to arrive at a better understanding of the differences between normal and abnormal color vision. It is hoped that further work of the kind described will indicate the nature of more reliable and discriminating tests of the kind which may be quickly administered.

[10] Forrest Lee Dimmick, Textile Colorist and Converter **67**, 114–115, 154 (1945).

[11] Arthur C. Hardy, *et al.*, *Handbook of Colorimetry* (The Technology Press, Mass. Inst. Tech., Cambridge, 1936).

[12] Committee on Colorimetry of the Optical Society of America. The following chapters have been published: Chap. II, J. Opt. Soc. Am. **33**, 544–554 (1943); Chap. V, J. Opt. Soc. Am. **34**, 183–218 (1944); Chap. VI, J. Opt. Soc. Am. **34**, 245–266 (1944); Chap. VII, J. Opt. Soc. Am. **34**, 633–688 (1944); Chap. VIII, J. Opt. Soc. Am. **35**, 1–25 (1945).

METHOD OF INVESTIGATION

When an object is illuminated with white light, the color perceived may be matched by illuminating a white body with a mixture of white light and spectrum light, provided that the color of the illuminated object does not fall in the area of purples. If the color lies in the excepted area, then (a) two spectrum lights (usually a red, the other a blue) must be admixed with the white light, or (b) the light from the object must be admixed with a single spectrum light of hue complementary to the color of the body to obtain a color matching the white body under the white illuminant.

In the present method the colored object may be any one of five deeply dyed papers having the following colors: red, orange, yellow-green, blue-green, and blue. The light reflected from one of the dyed papers is mixed in an integrating cavity, and the mixed light is seen by the subject through an aperture in the cavity wall. Immediately contiguous with the aperture is a second aperture in an adjacent integrating cavity, in which white light is mixed with spectrum light. The subject compares the light from this second aperture with that issuing from the first; if he perceives a difference, he may improve the color match by modifying (1) the light intensity in either cavity, (2) the wave-length of the spectrum light, and (3) the relative intensities of the white and spectrum lights.

By means of a procedure which is described in detail in a later section of this paper, the chromaticity (i.e., hue and purity) of the mixture is

TABLE I. I.C.I. colorimetric data for the colors to be matched (Illuminant *A*).

Color	x	y	Dominant wave-length, mμ	Purity* %	Y, %
Red	0.635	0.331	620	77.1	27.7
Orange	0.622	0.362	604	89.3	56.0
Yellow-green	0.415	0.513	561	49.5	57.3
Blue-green	0.334	0.485	507	25.8	31.4
Blue	0.322	0.360	488	31.8	24.1

* Excitation purity.

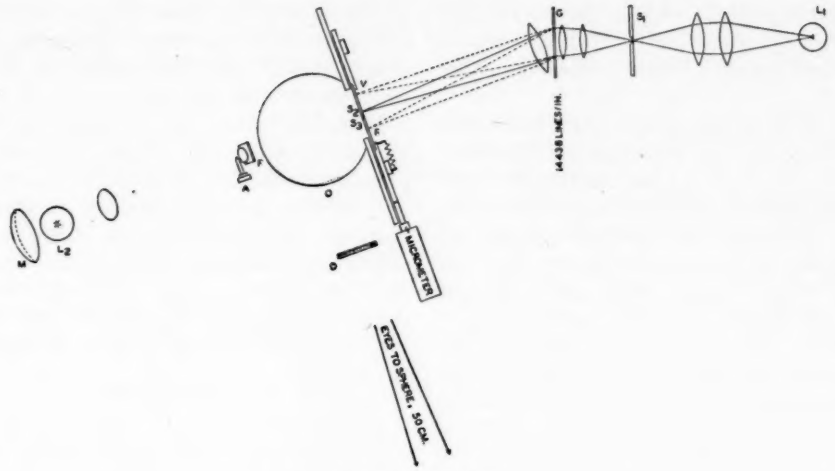

FIG. 2. Plan view of color-blindness apparatus.

plotted immediately on a large I.C.I. chromaticity diagram.* The supervisor of the test is thus enabled, during the actual period of testing, to map the chromaticities of the mixtures which the subject perceives as matches of the given color. The supervisor attempts to determine extrema on the diagram, making special note of each least perceptible difference (*LPD*). After a few points representing *LPD*'s have been mapped, the progression in purity and wave-length settings proceeds at a reasonable rate. Ordinarily, the wave-length is set at a fixed value and the purity is adjusted from too high and too low values to those corresponding to *LPD*'s. The wave-length is then changed to a neighboring value, and the process continued. When a match cannot be

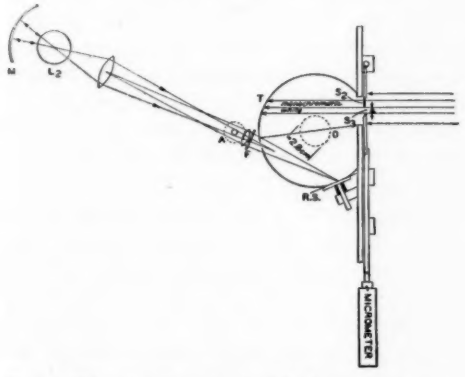

FIG. 3. Side elevation of color-blindness apparatus.

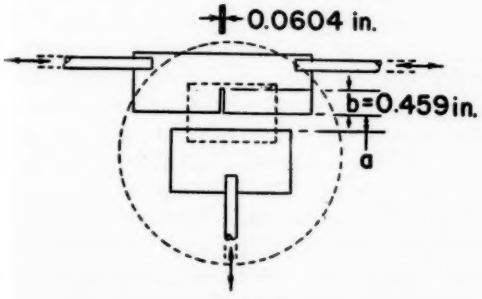

FIG. 4. Slit mechanism.

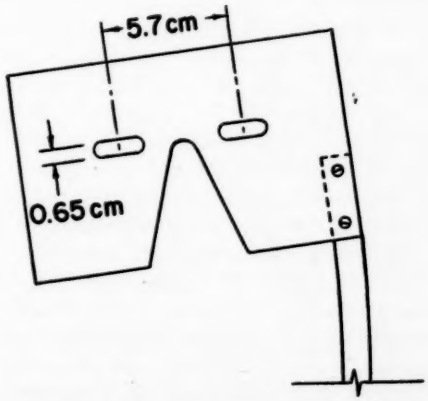

FIG. 5. Mask.

* The chromaticity is that which the I.C.I. Standard Observer would perceive for the mixture.

obtained for any purity setting, the wave-length is beyond an extremum, and the wave-length is modified until a match becomes possible for the subject.

In most of the recent work the subject varies only light intensity, and is not aware of numerical values or of the nature of the changes in wave-length and purity made by the supervisor. This procedure eliminates the detraction from the visual task of a three-valued adjustment which is unfamiliar to the average subject. Some success may be attained when the wave-length is fixed by the supervisor and the subject is asked to vary both light intensity and purity, but many of the subjects unfamiliar with the principles of colorimetry are confused by this two-valued adjustment; for example, (a) a change in purity modifies the luminosity of the color, and the unskilled observer may thereby confuse small changes in purity with small changes in light intensity, and (b) in any adjustment more complicated than a single-valued one, the unskilled observer may produce excessively large changes in one or more of the variables and thus miss combinations which would result in satisfactory matches. However, the most important aspect of the present procedure is that the supervisor can so direct the changes in wave-length and purity that a chromaticity limen will be obtained. The limen tends to correspond more to a "just noticeable difference" than to a perfect match.

The subject is required only to say that a match can, or cannot, be obtained (through adjustment of light intensity) following the changes in wave-length or purity produced by the supervisor. However, the subject is encouraged to describe, where possible, the nature of a color difference which he perceives. Subjects may use such expressions as "bluer than" or "yellower than" in describing a perceptible difference which is seemingly a huge difference. Subjects familiar with color nomenclature may describe a purity difference with such expressions as "too saturated" or "purity too high," but subjects unfamiliar with color nomenclature may have difficulty in properly describing a purity difference. It is often helpful to ask such subjects to think of colored solutions which are too concentrated or too dilute in comparison with a standard solution; such subjects may then use such phrases as "too weak," "too strong," or "too concentrated," and thus convey a proper description of what they see.

When the supervisor has completed the limen on the chromaticity diagram, the color to be matched is changed, the next limen is established, and the process continued until the limens have been determined for the previously mentioned colors—red, orange, yellow-green, blue-green, and blue. The spectral reflectivity curves for these dyed papers are presented in Fig. 1, and the I.C.I. colorimetric data are given in Table I.

APPARATUS

Schematic drawings of the equipment are presented in Figs. 2, 3, 4, and 5. Figure 2 shows a plan view of the monochromatizing system and the relationship between that system and the integrating cavities, the direction of viewing, and the system for illuminating the colored reflecting surface with heterogeneous light. The last named system and the integrating cavities are more clearly represented in Fig. 3. Figure 4 presents a drawing of the slit mechanism, as seen from the monochromatizing system, and Fig. 5 shows the fixed mask employed to orient the eyes and maintain their distance from the integrating cavities at a constant value.

As shown in Fig. 3, the integrating cavities are hemispheres, formed by dividing a 4-inch sphere with a plane partition. All inner surfaces of each cavity are coated with a thick layer of magnesium oxide. The lower cavity is filled with the light to be matched, and the upper cavity is filled with the mixture of white light and spectrum light. A circular opening (O) in the wall of the sphere enables the subject to compare the colors of the light in the two cavities. The light in the upper cavity is a mixture of tungsten filament light (previously referred to as "white") and spectrum light of wave-length L. The former is constituted of the complete spectrum light passing through the horizontal slit (S_3) of vertical opening a, and the latter is the spectrum light selected by the vertical slit (S_2), which has a length of 0.459 inch and a width of 0.0604 inch. The light rays through both slits strike the interior white wall in a target area (T), from which the light is reflected to all other portions of the cavity. The

light in the lower cavity is that reflected from the reflecting surface *R.S.* when the latter is illuminated with tungsten filament light originating in the lamp L_2. The reflecting surface may be magnesium carbonate (for standardizing adjustments), or one of the deeply dyed papers employed in the actual testing of a subject.

The monochromatizing system is comprised of a ribbon-filament projection lamp L_1 (Fig. 2) rated at 108 watts, light from which is condensed upon a narrow slit (S_1) by a pair of aspheric lenses. After collimation, the light is dispersed by a Welch replica projection grating (*G*) having 14,438 lines per inch. This grating was selected from a class designed to project a large fraction of the incident energy in the first-order spectrum. A grating rather than a prism was employed to obtain a nearly linear relationship between wavelength and slit position. The complete visible spectrum from violet (V) to red (R) falls, as shown, on the slit mechanism. Spectrum light of desired hue is obtained by horizontal motion of the slit S_2, which is actuated by the micrometer shown in Fig. 2. The amount of white light is controlled by adjustment of the micrometer shown in Fig. 3. The vertical extent of the spectrum falling on the slit mechanism is roughly three times the typical distance from the lower jaw of the horizontal slit to the top of the vertical slit; the intensity of light obtained is adequate, and avoidance of light originating near the ends of the ribbon filament minimizes any instrumental error arising from the fact that the temperature of the filament is distinctly lower near its ends than in its middle zone.

An adequately high light intensity in the lower cavity is obtained through the use of a condensing lens of short focal length, and a spherical mirror (*M*) (Fig. 3). Lamp L_2 is of the concentrated filament type, rated at 50 watts when operated at 6 volts. Light intensity is adjusted by means of a pointed screw (*A*), which enters and intercepts the beam of light where the latter is narrow. An amber-colored glass filter (*F*) corrects for the fact that the color temperature of lamp L_2 is considerably higher than that of lamp L_1. Small differences in color temperature are eliminated through adjustment of the voltage on lamp L_2, which is typically 5.7 volts. Light enters the lower cavity through a small aperture shown in Fig. 3 at a point just down beam from the filter *F*. The edges of this aperture are defined by thin black paper to obviate entry into the lower cavity of white light reflected from the (white) edge of the hole in the cavity wall; such light would produce a spuriously low purity of the light in the cavity.

The order of luminance of the field presented to the subject can be judged from the observation that, with magnesium carbonate in place as *R.S.* and the light intensity adjusted at the maximum setting, the light issuing from the lower half of the aperture *O* has a luminance of about 5.3 millilamberts. When *R.S.* is one of the colored papers, the maximum luminance would be the foregoing figure multiplied by the luminous reflectance of the paper (Y/100—*see* last column of Table I). Depending on the nature of the colored paper, the setting of S_2 and S_3, and the degree of abnormality of color vision of the subject, the actual luminance of the field may vary from values approaching the maximum for the given colored paper to quite low values. In the case of subjects having normal color vision, the adjustment of light intensity is nearly the same for all five colored papers, and the luminance is estimated to be roughly 5(Y/100) millilamberts.

The viewing arrangement (Figs. 2 and 5) provides for binocular vision of a divided circular field of 3.2 degrees. The surrounding field has low luminance. The eyes of the subject are screened from bothersome stray light, and the arrangement is designed to be free of error due to stray light entering the aperture in the sphere. The coarse dividing line between the semicircular fields formed by the metal partition bisecting the sphere is eliminated through the use of a pair of small-angle deviating prisms (*D* in Fig. 2). These prisms are mounted with their vertex edges in contact; the latter are finely ground, so that the only dividing line seen by the subject is the very narrow zone of contact between the two prisms. As a result of the displacement of the images of the two halves of the field, the shape of the field as actually perceived by the subject is not quite circular.

STANDARDIZATION OF THE APPARATUS

Immediately before and after a series of tests, the supervisor (a person having normal color

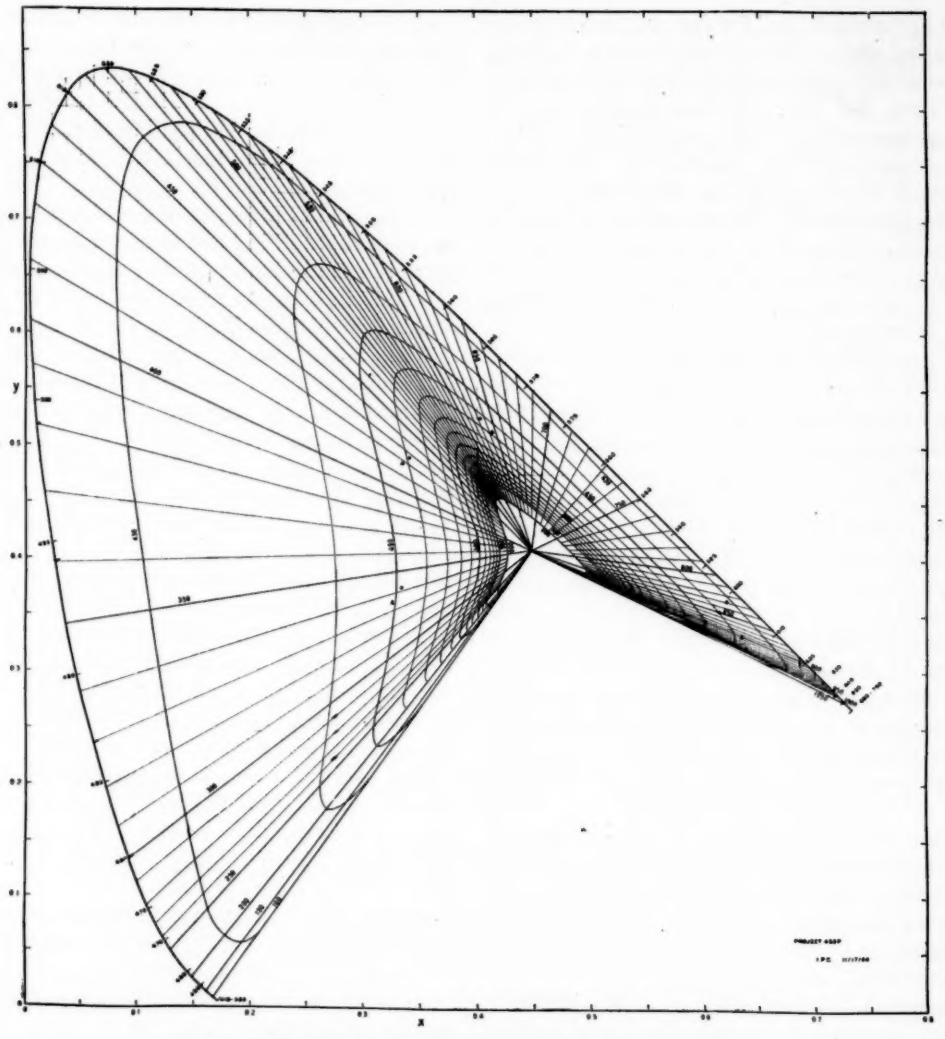

FIG. 6. Chromaticity diagram with special coordinate system.

vision) performs standardizing adjustments of the equipment. These consist of an adjustment for white light match, and a check of the wave-length scale. The former is accomplished by covering the vertical slit (S_2) and opening the horizontal slit (S_3) to a suitable value. The voltage on lamp L_2 and the intensity of the light are adjusted to obtain a white light match of the two fields.

The wave-length scale is checked through the use of a steno-chromatic filter (Wratten 73) placed between filter F (Fig. 3) and the entrance aperture of the lower cavity. The horizontal slit is closed, the vertical slit is uncovered and adjusted to obtain a match of the two fields, and the micrometer is read. (The intensity of the spectrum light is reduced to a suitable value through the use of an auxiliary screen placed in the monochromatizing system.) After a warm-up period, the average of several micrometer settings is ordinarily found to be close to a certain long-time average value, which is taken as standard;

the difference is added to or subtracted from the wave-length readings subsequently obtained for the subject under test. In both of these adjustments, the reflecting surface *R.S.* is magnesium carbonate.

The choice of the Wratten 73 filter was based primarily on the consideration that the effective wave-length of light passed by this filter and originating in a tungsten filament lamp of color temperature approximating that of I.C.I. Illuminant *A* is about 578 mμ, for which value the eye is very sensitive to changes in wave-length. The use of the nearly monochromatic light passed by a Wratten 73 filter renders negligible any systematic error produced by differences in color vision in the same normal observer from day to day, or in different normal observers, because the only requirement for the accurate matching of two fields of spectrum light is that the observer shall be sensitive to hue changes in the wave-length range of interest.

DETERMINATION OF CHROMATICITY

The chromaticity of a color is comprised of *hue* and *purity*. When the luminous reflectance of a solid colored body, or the luminance of a beam of light is also given, the color becomes completely specified. Thus, the chromaticity is that part of color which enables one to differentiate with respect to hue (green, or yellow, or blue, etc.) and purity of the color (saturation). The chromaticity is numerically expressed in terms of trichromatic coefficients (x, y) which, when plotted on an I.C.I. chromaticity diagram, enable one to express the chromaticity in terms of *dominant wave-length* and *purity* (relative to a given illuminant). In the present work, the reference illuminant is tungsten filament light approximating I.C.I. Illuminant *A*.

The foregoing may be illustrated with the yellow-green paper (*see* third row of Table I). The trichromatic coefficients for light reflected from this paper (incident light Illuminant *A*) are seen to be 0.415 and 0.513. These coefficients are plotted on an I.C.I. chromaticity diagram in Fig. 6. The point, represented by a small triangle, is seen to lie on a straight line joining the "white point" for Illuminant *A* (the point of convergence of the radiating lines) and the point on the spectrum locus corresponding to monochromatic light of wave-length 561 mμ. The significance of this is that the chromaticity of the light reflected from the yellow-green paper may be matched by combining, in proper proportions, the yellowish-white light of Illuminant *A* and spectrum light of wave-length 561 mμ—if the match is performed by the (hypothetical) I.C.I. Standard Observer. Thus, the figure of 561 mμ is the dominant wave-length of the color, with reference to Illuminant *A*. The proportion of white light and spectrum light determines the purity. In this article, excitation purity is employed. This quantity is the fraction of the total distance from the white point to the point on the spectrum locus which is covered by the interval from the white point to the point giving the chromaticity of the light reflected from the yellow-green paper; it is expressed as a percentage. The purity for the yellow-green sample is found to be 49.5 percent.

The chromaticity diagram shown in Fig. 6 is furnished with a special, auxiliary coordinate system, the purpose and use of which will be discussed later.

The equipment developed for this investigation does not evaluate chromaticity with the high absolute accuracy required for color measurement, since good reproducibility or precision was felt to be the major object in designing an apparatus to be used only for the study of abnormalities of color vision. The principal interest in the present study lies in the determination of the approximate shape, size, and disposition of chromaticity limens for subjects of abnormal color vision, as compared with those for persons of normal color vision. It is clear, then, that only fair absolute accuracy with good precision is required, and that the substantially greater expense of constructing a highly accurate monochromatic colorimeter would not be justified at this time. The following method for determining chromaticity was evolved with these considerations in mind.

For convenience in calculating chromaticities, it is assumed that the white light in the cavities during a "white light match" has the relative spectral energy distribution of I.C.I. Illuminant *A*. Actually, the white light is slightly but perceptibly yellow-green when compared with an adjacent field of light from a standard Mazda 100-watt lamp operated at normal voltage. How-

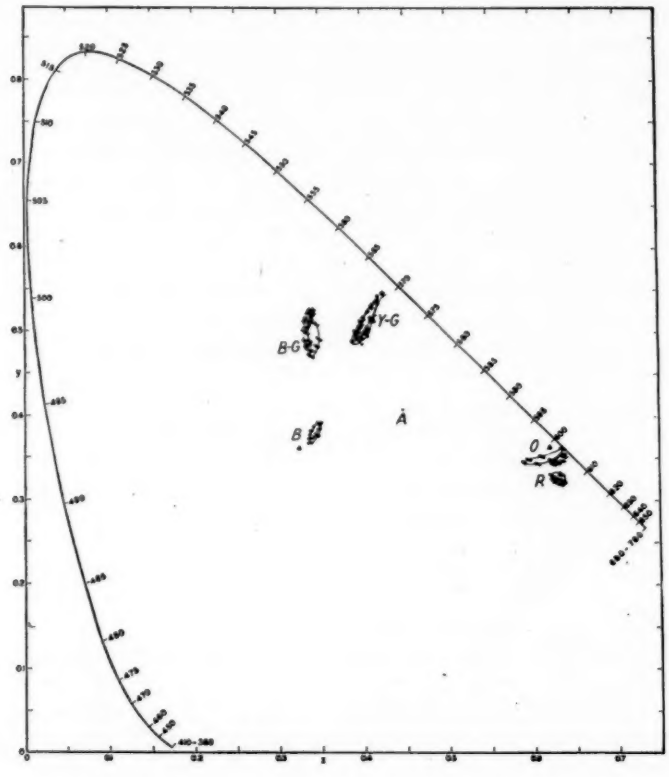

FIG. 7. Limens for AB.

ever, the color of the white light is maintained constant within small limits.

The dominant wave-length is measured directly. An exact equation relating distance between the vertical slit and the position of the slit corresponding to the standardizing setting (577.8 mμ) reveals that little accuracy is lost in assuming a linear relationship. The wave-length error toward the ends of the spectrum is of the order of one millimicron, but is constant and, hence, does not influence accuracy of comparison between individuals. To simplify the conversion from readings on the "wave-length micrometer" to the wave-length expressed in millimicrons, a pair of adjacent, sliding scales one meter long was prepared. These scales were prepared from a pair of meter sticks. It developed that the millimeter divisions on one stick could be used directly as thousandths of an inch. A paper scale for wave-length was carefully prepared on the basis of the linear relationship referred to above, and this scale was bonded with rubber cement to the second meter stick. The standardizing wave-length is indicated on the latter scale by a line drawn at 577.8 mμ. After the apparatus has attained thermal equilibrium, the micrometer

TABLE II. Responses of seven observers to the Ishihara test for color blindness.

Observer		AB	BC	CD	DE	EF	FG	GH
Sex		M	M	F	F	M	M	M
Age		47	40	40	24	32	36	27
Plate	1	12	12	12	12	12	12	12
	2	8	8	8	8	*3*	*3*	*3*
	3	6	6	6	6	6	*5*	*5*
	4	5	5	5	5	*2*	*2*	*2*
	5	74	74	74	74	*21*	*21*	*21*
	6	2	2	2	2	*B*	*B*	*B*
	7	6	6	6	6	*B*	*B*	*B*
	8	5	5	5	5	*B*	*B*	*B*
	9	7	7	7	7	*B*	*B*	*B*
	10	B	B	B	B	*5*	*5*	*5*
	11	B	B	B	B	*2*	*2*	*2*
	12	26	26	26	26	2*B*	2*B*	26
	13	42	42	42	42	4*B*	4*B*	4*B*
	14	T	T	T	T	*B*	*B*	*B*
	15	T	T	T	T	*B*	*B*	*B*
	16	B	B	B	B	*T*	*T*	*T*

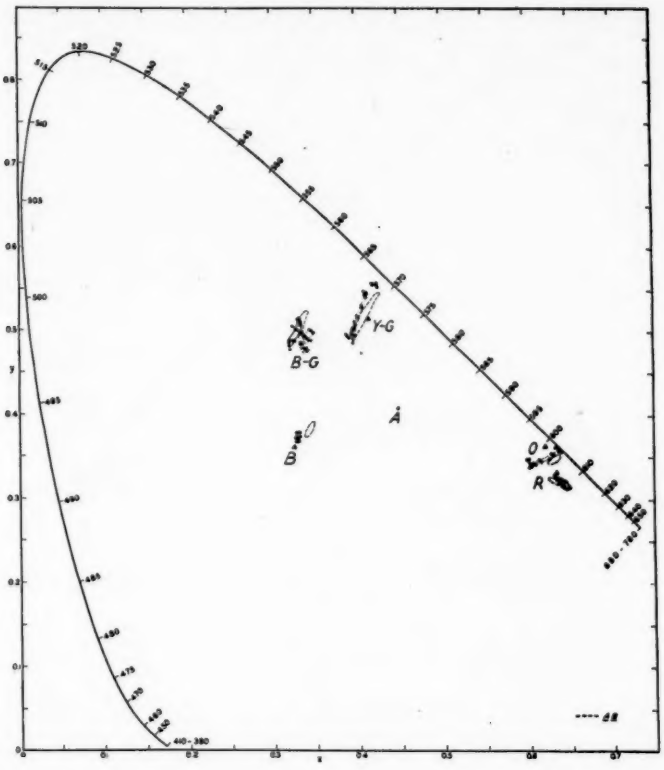

FIG. 8. Limens for BC (for comparison, limens for AB are included).

reading obtained during the wave-length standardization (*see* earlier section on "Standardization of the Apparatus") is usually close to 718 mils (or 0.718 inch). The micrometer scale is adjusted so that this reading falls on 577.8 mμ. The chart shown in Fig. 6 is based on this micrometer reading. If, at any particular testing time, the micrometer reading obtained in the wave-length standardization is different from this value, a simple correction is applied to all the subsequent micrometer readings; the correction which is added to the micrometer readings is $(718-m_L)$, where m_L is the micrometer reading (in mils) obtained in the wave-length standardization. In the procedure now employed for determining the location of a point on the chromaticity diagram, this corrected micrometer reading is employed directly, and the conversion scale to determine dominant wave-length is used only in special cases when, for some reason, it is desired to know this quantity for a given micrometer setting.

The opening of the horizontal slit, which controls the purity of the mixture of light in the upper cavity, is obtained directly from the reading of the purity micrometer; this opening (a), expressed in inches, is simply the micrometer reading less the zero setting. On the assumption that the vertical width of the spectrum formed in the apparatus is constant, and that the distribution of light in the vertical direction is uniform over a distance not greater than the maximum value of a, the tristimulus values X, Y, and Z of the white light passing through the horizontal slit are proportional, respectively, to 11,850a, 10,790a, and 3840a (*see*, e.g., the M.I.T. *Handbook of Colorimetry*[11]). As previously discussed, this is subject to the error of the assumption that the white light is I.C.I. Illuminant A. The width of the wave-length band passed by the vertical slit, for zero width of the slit S_1 (Fig. 2), is 14.14 mμ.* If the vertical dimension of the wave-

* Since the slit S_1 is actually finite, one may think of the light passing through the vertical slit as a continuous succession of overlapping bands of light, each 14.14 mμ

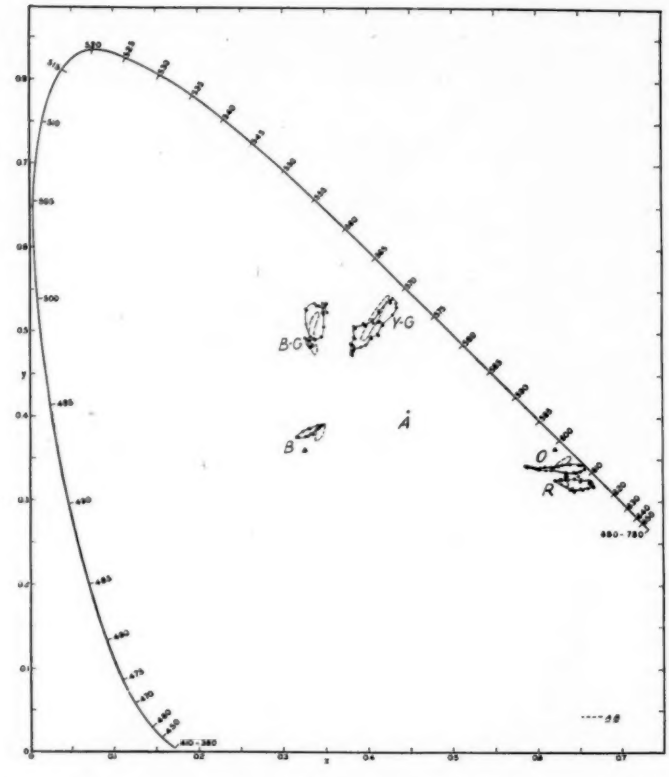

FIG. 9. Limens for CD (for comparison, limens for AB are included).

length slit is b, the trichromatic coefficients of the mixture of white and spectrum light are:

$$x=[11850a+14.14b(E_A\bar{x})]/D, \quad (1)$$

$$y=[10790a+14.14b(E_A\bar{y})]/D, \quad (2)$$

where

$$D=26480a+14.14b[(E_A\bar{x})+(E_A\bar{y})+(E_A\bar{z})]. \quad (3)$$

Substituting for b its value, 0.459 inch, and dividing both numerator and denominator of Eqs. (2) and (3) by $14.14b$, we obtain:

$$x=(1825a+E_A\bar{x})/D', \quad (4)$$

$$y=(1663a+E_A\bar{y})/D', \quad (5)$$

$$D'=4080a+E_A\bar{x}+E_A\bar{y}+E_A\bar{z}. \quad (6)$$

In the foregoing Eqs. the products $E_A\bar{x}$, etc., are conveniently obtainable from Table XIII of the M.I.T. *Handbook of Colorimetry*.[11]

In the earlier phases of the work reported herein, Eqs. (4), (5), and (6) were used to calculate chromaticities corresponding to the micrometer settings for a given color match. Although the calculations were facilitated by an automatic calculating machine, it soon became evident that a conversion chart would have two very desirable characteristics: (1) Its use would save time in the conversion of micrometer readings to chromaticity. (2) With a copy of the chart at hand *during the testing* of a subject, it would be possible to explore more effectively the chromaticity limens. Experience with the chart shown in Fig. 6 has proved the latter characteristic to be invaluable, particularly where the subject has abnormal color vision; where the limen for such an observer is extensive and of peculiar shape, the supervisor finds the chart of immense help in filling areas where data are inadequate. After a number of matches have been

wide. However, since the excitation functions for the eye do not change rapidly with wave-length over small ranges, the chromaticity calculated on the basis of zero slit width will be substantially in agreement with that calculated by more rigorously correct methods.

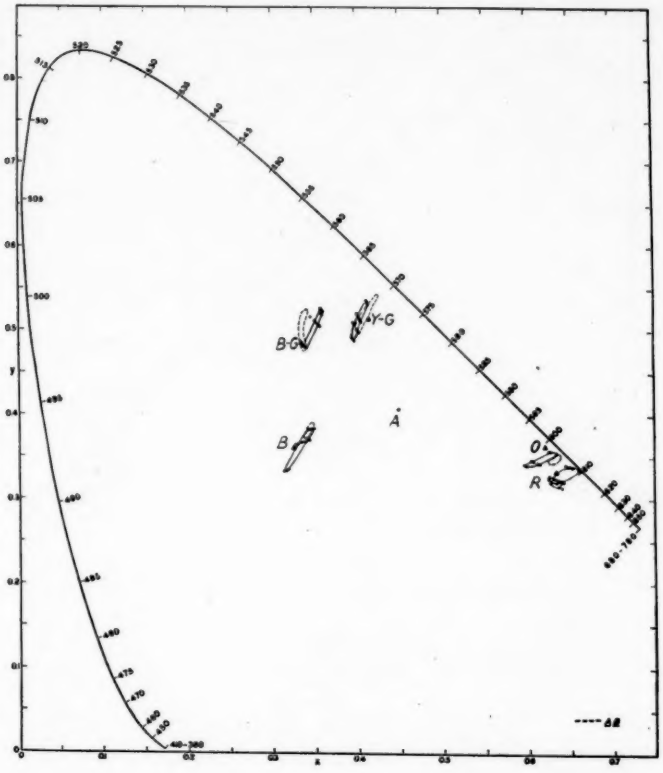

FIG. 10. Limens for DE (for comparison, limens for AB are included).

obtained, it is even possible to forecast roughly the probable course of the limen and, hence, much time is saved in avoiding instrumental settings which might be far removed from the limen. This saving in time, combined with the elimination of an interval between the recording of instrumental settings and calculation of chromaticities through Eqs. (4), (5), and (6), has resulted in a considerable lessening of the over-all time required to establish the limens for a given subject on the I.C.I. chromaticity diagram.

With reference to Fig. 6, lines of constant wave-length micrometer settings are straight lines running from the point for Illuminant A to the spectrum locus. The lines for constant settings of the purity micrometer will be seen to run around the diagram in a manner roughly paralleling the spectrum locus. These latter lines are *not* lines of constant excitation purity, and should not be confused with constant purity loci. The chart was prepared by calculating chromaticities corresponding to a number of cardinal settings of the two micrometers. Except for mentally applied corrections to the micrometer readings (add $718-m_L$ to the wave-length micrometer, as previously discussed, and add a correction to the purity micrometer in the event that zero settings have changed), the micrometer readings are used directly on the chart.

A 16×18-inch positive photostat copy of the chart reproduced in Fig. 6 is employed in the laboratory for each subject. Because of the difficulty of reproducing these photostat copies on a reduced scale, the charts shown in Figs. 7 to 13 have been especially prepared for publication, and do not include the lines for constant wave-length and purity micrometer settings.

THE INFLUENCE ON LIMENS OF THE METHOD OF VARIATION OF LUMINANCE

Complete representation of colors requires a three-dimensional coordinate system. The trichromatic coefficients x and y are two of the dimensions, and the luminance is the third.

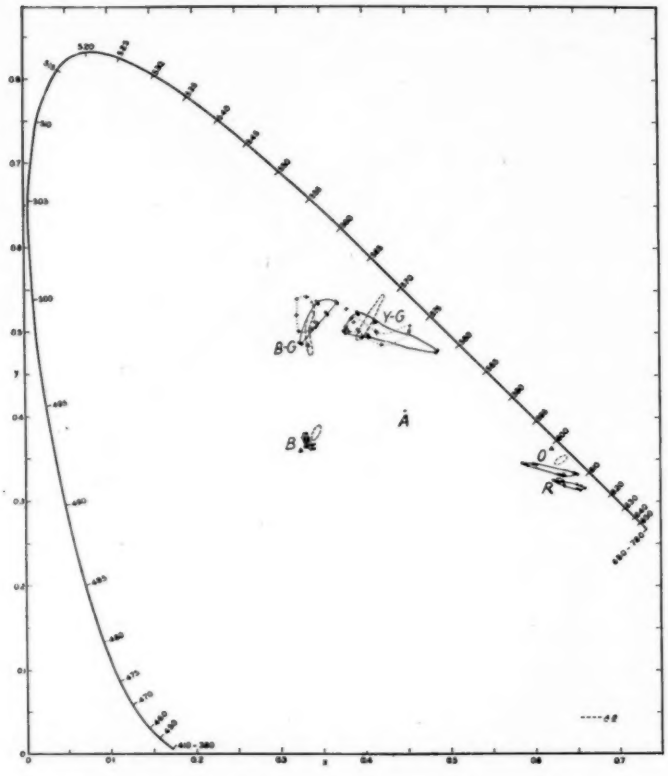

FIG. 11. Limens for EF (for comparison, limens for AB are included).

Ordinarily, one visualizes x and y as being plotted horizontally, and luminance vertically. The chromaticity diagram is then the base of the color solid; a variation perpendicular to the base is one simply of luminance, or, in more popular terms, of lightness or darkness.

The limens of greatest ultimate interest are closed, curved surfaces lying in the color solid existing in the three-dimensional system described above. For the present, interest centers in the determination of the *projection* of the three-dimensional limens on the chromaticity plane. The shape and orientation of three-dimensional limens throughout the color solid have not yet been determined even for normal subjects. Some writers have supposed that the surfaces are ellipsoidal, and others have imagined the still more restricted picture of ellipsoidal surfaces oriented with a major axis parallel to the axis of luminance (vertical axis). There appears to be no *a priori* reason for supposing that the surfaces for normals have such simple shapes as ellipsoids, or that the simple shape is oriented with a principal axis in the vertical direction; in fact, the charts presented later furnish evidence to the contrary.

One method for determining the three-dimensional limens would consist in holding the luminance of the color to be matched (in the lower cavity) at a constant value, and in varying the chromaticity and luminance of the mixture of white and spectrum light (in the upper cavity). In the present method, the luminance of the color to be matched was varied in the interest of obtaining as high a light level in the matching as possible. In consequence, the three-dimensional limen is to be visualized as moving up and down during the testing of the subject. If the limen should remain constant in size, shape, and orientation, then the chromaticity limen finally appearing on the chromaticity diagram would be a true projection of the three-dimensional limen. It is known, however, that sensibility of the eye depends upon luminance and, hence, the movement of the three-dimensional limen involved in

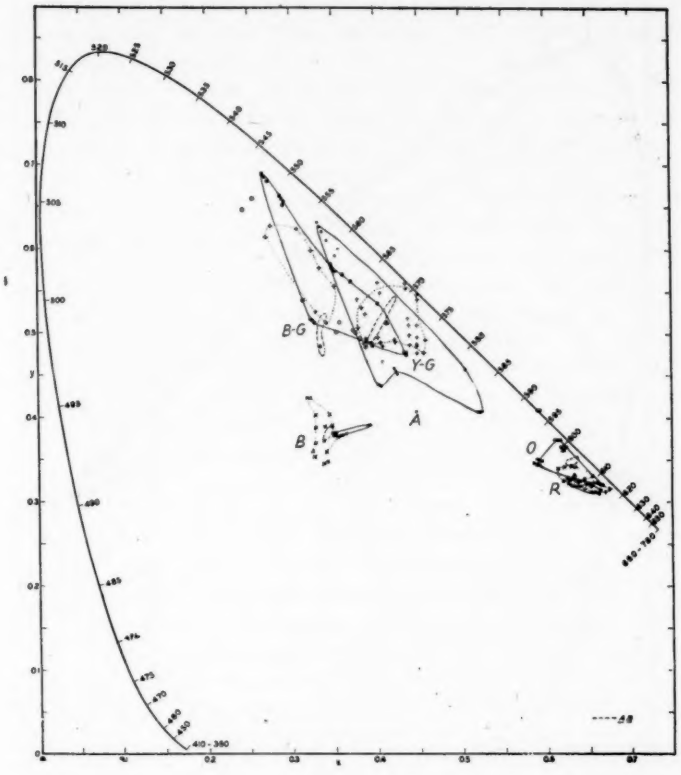

FIG. 12. Limens for FG (for comparison, limens for AB are included).

the present method would produce a distortion in the projection. However, the variation in luminance is small, and it seems probable that this distortion is not great.

Where the subject has strongly abnormal color vision, and the area of match covers an extensive region of the chromaticity diagram, the greater variations in luminance brought about by the large variations in dominant wave-length and purity of the matching light would undoubtedly produce greater distortions in the observed chromaticity limens.

It should be recalled, however, that the main objective in this work is the determination of the approximate shape, size, and disposition of chromaticity limens for tested subjects, relative to the limens for persons of normal color vision. It is not believed that the above-mentioned distortions in the projections of the three-dimensional limens will influence the validity of the conclusions drawn in this study. The distortions are discussed here primarily by way of pointing out the desirability of eventually modifying the instrumental method to make it suitable for the determination of the complete limens in the color solid.

USE OF THE SPECIAL CHROMATICITY CHART

The special chromaticity chart which was prepared to facilitate the plotting of observations has been described and is shown in Fig. 6. The data for each observer have been plotted on a single chart of this kind (see Figs. 7 to 13). These charts have been simplified to make them easier to read in reduced form in this publication.

The reader's attention is directed to the relative location, size, shape, and orientation of areas of match enclosed by the lines representing the limens of the observer.

The spectrophotometric descriptions of the colored samples, as seen by the I.C.I. Standard Observer and given in Table I, are indicated by small triangles on the charts.

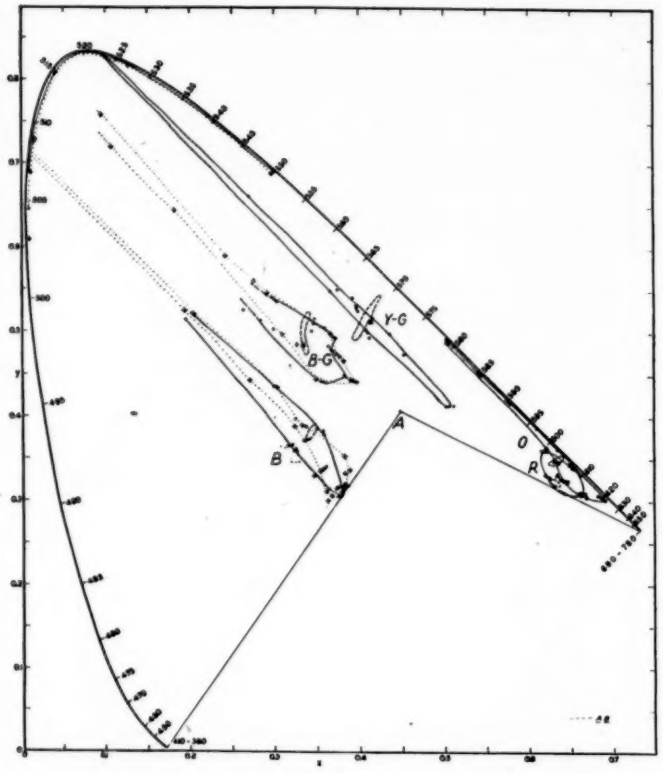

FIG. 13. Limens for GH (for comparison, limens for AB are included).

THE OBSERVERS

The data presented in the seven charts (Figs. 7 to 13, inclusive) are for seven observers, two of whom are women and who were included because women are known usually to possess normal color vision. The responses of the observers to the Ishihara tests for color blindness (5th edition, 1932), the American Optical Company pseudo-isochromatic plates for color blindness (1940 edition), and the Glenn Colorule (Research Laboratory, Sidney Blumenthal & Company, Inc., Shelton, Conn.) are given in Tables II, III, and IV.

In Tables II and III, plate numbers refer to the individual plates used for the first two tests. The digits or letter (*H*) opposite the plate numbers are the figures (or letter) read by the observer. A figure which is italicized represents a deviation from the normal response; the letter B indicates that the observer saw no figure or configuration; *B* indicates that the observer saw no figure when the normal observer did see a figure. A *B* following a figure indicates that the observer could see only one (or two) of the two (or three) digits comprising the number on the plate. The letter T is used to indicate the configuration on a plate which can be seen by the normal but not by the color-blind observer; *T* represents the reverse situation.

The Glenn Colorule consists of two slides with patches of dyed textile on each slide. The patches on the upper slide are designated by figures and those on the lower slide by letters. The combinations of figures and letters in Table IV indicate the matches selected by the observer; for example, BC matched color patch 7 of the upper slide with color patch E of the lower slide.

INDIVIDUAL RESULTS

AB (male, age 47) reacted normally to the pseudo-isochromatic tests and by comparison with others of normal color vision, to the Glenn Colorule (see Tables II, III, and IV). His limen

for each sample (see Fig. 7) was near the point representing the chromaticity of the sample as seen by the I.C.I. Standard Observer, which is represented by a small triangle with the letters designating the color of the sample. AB's limens consistently tend to fall at a longer wave-length and lesser purity than the points of the I.C.I. Standard Observer, except for the yellow-green sample. These differences are discussed in a later section of this article.

The size of the area of match varied on the chromaticity diagram from sample to sample, the blue and red being the smaller and the blue-green and yellow-green the larger.

The two limens shown in Fig. 7 for each color represent the results of observation on two different days. The solid line which is drawn through the dots indicates the first series of observations. The dotted line which is drawn through the plus signs represents the second series. This representation is used in Figs. 7–13. It is clear that the size of the second area was smaller than that of the first for each color. For the blue-green and orange colors, the location changed a little toward greater purity. For the other samples, the location remained much the same. The limen for the blue-green color was more elongated in the second series.

No change in the instrument is associated with the changes just pointed out. A change in procedure accounted for the change in shape of the limens but not for any change in size or location. The shapes were not comparable because, in the first series, the points were obtained and calculated before the special chart had been prepared, and only maximum and minimum wave-lengths with their corresponding purities and two

TABLE III. Responses of seven observers to the A.O.C. pseudo-isochromatic plates for testing color perception.

Observer		AB	BC	CD	DE	EF	FG	GH
Sex		M	M	F	F	M	M	M
Age		47	40	40	24	32	36	27
Plate	1	89	89	89	89	*B*	*B*	*80*
	2	43	43	43	43	*B*	43	43
	3	56	56	56	56	*B*	*B*	*36*
	4	27	27	27	27	*B*	*2B*	*22*
	5	8	8	8	8	8	8	*3*
	6	6	6	6	6	6	*5*	*5*
	7	39	39	39	39	*B*	*B*	*B*
	8	42	42	42	42	*B*	*B*	*B*
	9	56	56	56	56	*B*	*B*	*B*
	10	27	27	27	27	*B*	*B*	*B*
	11	29	29	29	29*	*20*	*70*	*70*
	12	57	57	57	57*	57	*35*	*35*
	13	86	86	86	86	86	*B*	*B*
	14	75	75	75	75	*B*	*B*	*B*
	15	7	7	7	7	7	7	7
	16	9	9	9	9	9	9	9
	17	25	25	25	25	*B*	*B*	*B*
	18	68	68	68	68	*B*	*B*	*89*
	19	5	5	5	5*	*2*	*2*	*2*
	20	3	3	3	3*	*5*	*5*	*5*
	21	97	97	97	97	*B*	*B*	97
	22	34	34	34	34	*B*	*B*	*35*
	23	56	56	56	56	*B*	*B*	*B*
	24	27	27	27	27	*B*	*B*	*B*
	25	12	12	12	12	12	12	12
	26	H	H	H	H	H	H	H
	27	89	89	89	89	*B*	*B*	*B*
	28	43	43	43	43	*B*	*B*	*B*
	29	86	86	86	86	*B*	*B*	*B*
	30	75	75	75	75	*B*	*B*	*B*
	31	52	52	52	52	52	*62*	*B*
	32	96	96	96	96	96	96	96
	33	B	B	B	B	B	*45*	*45*
	34	B	B	B	B	B	*21*	*23*
	35	T	T	T	T	T*	*B*	*B*
	36	T	T	T	T	*B*	*B*	*B*
	37	052	052	052	052*	*B*	*B*	*B*
	38	394	394	394	394*	*B*	*B*	*B94*
	39	23	23	23	23*	*B*	*B*	*B*
	40	65	65	65	65*	*B*	*B*	*B*
	41	15	15	15	15	15	17	17
	42	74	74	74	74	*71*	*21*	*24*
	43	47	47	47	47*	*B*	*B*	*B*
	44	98	98	98	98*	*B*	*B*	*B*
	45	B	B	B	B	B	*T*	*T*
	46	T	T	T	T	T	T	T

* Observer evidenced difficulty in reading this plate.

TABLE IV. Responses of seven observers to the Glenn Colorule.

Observer	BC	CD	DE	EF	FG	GH	
				Daylight			
Observer	7	8	7	16	19	6	21*
	E	F	E	M	P	A	P*
AB (Control observer)**	7	7	7	11	8	7	7
	E	E	E	G	G	E	E
Difference	0	1	0	5	11	−1	14
	0	1	0	6	9	−4	11
				Artificial Light			
Observer	17	17	16	23	22	24	
	R	R	P	V	V	V	
AB (Control observer)**	17	17	16	16	16	16	
	R	R	Q	Q	Q	P	
Difference	0	0	0	7	6	8	
	0	0	1	5	5	6	

* Observer considered this match practically as good as 6-*A*.
** Not all results were taken under the same illuminant; hence, the data for the control observer were not constant.

bearings across the area of match at $\frac{1}{3}$ and $\frac{2}{3}$ of the total wave-length interval were made to determine purity extrema. These gave the long axes and two cross dimensions. In the later series, made after the conversion chart had been completed, many more points were directly plotted, which made it possible to sketch in the shape of the limens more accurately.

The observer was aware of the improved discrimination in making matches as he became more experienced and, accordingly, was able to distinguish more closely between differences of hue, purity, and luminance to the extent that the areas of match were noticeably smaller for the later observations.

The changes in location may have been due to differences in the physical condition of the observer.

BC (male, age 40) responded normally to the Ishihara, American Optical Company, and Glenn Colorule tests (*see* Tables II, III, and IV).

Figure 8 shows that the location, size, shape, and orientation of BC's limens varied for the different colors in very much the same way as did those of observer AB and that the limens were very similar in location, size, shape, and orientation to those of AB, although some noticeable differences were apparent. The limens for the last series of observations of AB are shown for comparison in Fig. 8 in lightly dashed lines. BC's limens for blue, orange, and blue-green were a little nearer the point of chromaticity of the I.C.I. Standard Observer than those of AB's and nearly the same distance away for the other two colors.

Observer BC made a second series of matches on all the samples except blue, which gave limens that were narrower and smaller and similar in location to his first series. The second blue-green series partially overlapped both limens of AB. Again, the exact shapes of the first and second series of observations cannot be compared because, in the first series, only the extreme wave-lengths with their corresponding purities were determined and one bearing across the area of match was made at the mid-wave-length to determine extremes in purity. These points were inadequate to determine the limens. In the second series, many points on the limens were plotted directly on the specially prepared chart.

BC showed differences in discrimination on different days—which appeared to have been due to experience and practice in relation to differences in the size of the limens and to physical condition in relation to the differences in location of the limens.

CD (female, age 40) responded normally to the Ishihara, American Optical Company, and Glenn Colorule tests (see Tables II, III, and IV). Figure 9 shows the same type of variation in location, size, shape, and orientation of the limens for the different colors as observed in the preceding data, but it is to be noted that the limens of CD were all larger than those of AB, which are sketched in this figure in light dashed lines for comparison.

Because all of the observations for CD were made after the special chart was prepared, only one set of limens is plotted. But note should be made that the discrimination of the observer improved with the experience of the hour or two in matching each color to the extent that rechecking observations at the end of the period reduced the size of the limen in the areas which had been explored at the beginning of the period.

The observer was encouraged to be more critical in accepting matches but, even so, the area of match for each color was larger and farther from the point of chromaticity of the color as seen by the I.C.I. Standard Observer than for AB.

Considerable care and time were taken in exploring the limens of this observer and special attention is called to the shape of the limens. While the areas tended to be elliptical, small tails or points were clearly indicated at the extremes; e.g., the blue-green color in the neighborhood of $x=0.33$, $y=0.48$. The writers are confident that these shapes represent definite characteristics of the color vision of this observer on the date the matches were made.

CD was evidently much interested in the color matching experience and expressed pleasure in participating in it. She showed very little fatigue, even though she worked steadily for periods of four hours at a time with only short rests during matchings when she would look away from the instrument for a few seconds.

DE (female, age 24) responded normally to the Ishihara (Table II) and to the Glenn Colorule

tests (Table IV) but showed difficulty in reading correctly several of the plates in the American Optical Company series (Table III). This indicated a slight red-green deficiency according to the authors of the test.

Figure 10 shows the limens for DE, the more significant aspects of which were the larger size and the different location of the limens in comparison with those for AB, which are also shown on the chart. For example, the limen for the blue color was larger, and included shorter wave-lengths with greater saturation. The larger size of the limen indicated a larger area of match—i.e., the observer was a little less sensitive to blue and red and, therefore, accepted chromaticities as matches over a larger extent of the color diagram than did AB.

This observer made only one series of matches, so there was no evidence of change due to experience or physical condition. She was encouraged to be careful in matching and to accept only those which were equal in hue, purity, and luminance. DE preferred to adjust both the purity and luminance herself in making her judgments.

The data presented are for observations which were calculated and plotted before the special chart was prepared. Consequently, each limen was located by the axis of the wave-length extrema and cross bearings at the $\frac{1}{3}$ and $\frac{2}{3}$ portions of the long axis. Thus, these observations located the extrema of wave-lengths, which give the length and orientation of the longer axis of the area of match, and the ranges of purity at two intersections of the long axis.

DE commented on the fading of color after retinal fatigue. She also manifested mild signs of restlessness and discomfort.

EF (male, age 32) read several of the Ishihara and American Optical Co. plates as do those who are partially red-green blind (see Tables II and III). He also chose matches on the Glenn Colorule several steps to the right of the normal observer (Table IV). According to the authors of the tests, these results show some deficiency in seeing green and red.

The significant aspects which appeared on EF's chart (Fig. 11) were the size, orientation and location of his limens. Those for the yellow-green and orange colors lay at a different angle and in a different location from the corresponding ones for AB.

EF completed the second series of matches for the yellow-green, blue, and blue-green colors. The limens of the second series were smaller than those of the first series of matches. This appeared to denote experience and more careful discrimination on the part of the observer and, perhaps, reduction of visual fatigue. In the first series, he often described the fading of color after a few minutes of observation, so that he could not distinguish differences which he could see again after a few minutes of resting his eyes by looking away from the instrument. He also described the process of matching as requiring much effort, and complained of eye strain. This observer manifested physical restlessness.

In the second series this observer rested his eyes often and the periods of matching were shortened to about one hour or less a day. Even with these special efforts to avoid fatigue, EF's limens for the blue-green and yellow-green were larger than for those of the normal observers and, more important, his limens were appreciably displaced from those of the normal observers. The second series for the samples of orange and red were not completed, because of the observer's reluctance to continue the testing at the time.

FG (male, age 36) responded to the pseudo-isochromatic charts as do those observers who are red-green blind (Tables II and III) and did not detect much of the green in the Glenn Colorule test (Table IV).

The first matches by this observer (Fig. 12) were made before he became ill and was absent from work for four weeks. He complained of his eyes being tired at the time and wanted to rest them frequently. He experienced fatigue—i.e., after looking at the color area for a few minutes, the colors would fade out and he would see no difference in the two halves of the matching sphere. After looking away for a minute or two for rest, he could see a difference in color which would again fade out after sustained observation.

When FG returned after his illness, the supervisor discussed the results of the first series of matches with him and a new series of matches was attempted. Difficulty was encountered because he was reluctant to accept any match. Lacking confidence in his sensitivity—especially

to green and yellow-green—FG always saw a difference or imagined he saw a difference in every combination presented to him. Matches which were acceptable in the first series did not satisfy him in the second series. Apparently on the defensive, FG would only grant that a match was "pretty good" or "fair," never "good" or a "match" on the blue, orange, red, and yellow-green samples. In testing FG the second time, the supervisor set many matches for these colors and the results indicated in Fig. 12 as the second series are a conservative consolidation of his consistent judgments regarding stimuli which were set to confuse him.

The final sample of the second series was blue-green. So much defensiveness was encountered that the method of testing was changed. He then indicated those settings which were clearly outside of the area of match (represented by the small open circles in Fig. 12) and also those in his region of recognized uncertainty.

FG complained of the lack of color in the green and yellow-green samples and commented on the richness of the color in the red sample in comparison with the others. Once when the supervisor was checking the white light match, he asked FG to look at it. FG said that it was a pink color, not a white light; later, when he was confronted by the blue-green color, he said that it was "white—whiter than that white you showed me a little while ago which I said was pink."

The areas of match for the yellow-green sample overlapped the area of match for the blue-green sample. A setting of the chromaticity in the overlapping portion of these two areas was accepted as a match for either the blue-green or the yellow-green sample.

The limens for AB are drawn in lightly for comparison.

GH (male, age 27) showed serious deviation from normal color vision on all the Ishihara plates and nearly all of the American Optical Company plates (Tables II and III) and in the Glenn Colorule test (Table IV).

Figure 13 presents the data for this observer. It will be noticed at once that the areas of match were large and long. Characteristically they all extend over wide ranges of both wave-length and purity. The directions of the long axes suggest the condition which describes a deuteranope (a green-blind person) according to Pitt.[13] However, these axes do not converge closely on the point $x=1.0$, $y=0$; they are more nearly parallel.

Figure 13 shows that GH accepted as a match for the blue color settings ranging rather narrowly from approximately 400 mμ and 25 percent purity to approximately 505 mμ and complete saturation—with a very interesting additional set of matches all the way to nearly 550 mμ at 100 percent purity. Note should be made of the enlargement of the area around the chromaticity of the sample as seen by the I.C.I. Standard Observer.

Similarly in matching the blue-green sample GH accepted settings ranging from approximately 14 percent purity to high purity, with enlargement of the area of match around the chromaticity of the color to be matched. Even longer was his limen for the yellow-green sample, which extended from approximately 523 mμ and 100 percent purity to approximately 592 mμ and 40 percent purity.

The area of match for the orange sample showed the same characteristics lying between 578 mμ and 640 mμ along the spectrum locus—i.e., with high purity.

Only the red area of match was noticeably different in shape from the others and even that had an orientation similar to the other areas.

The limens for AB are drawn in Fig. 13 for comparison. The limens for GH and AB represent the greatest contrast in the data presented in this report. The differences in chromaticity which GH accepted as matches appeared very great to AB, whereas the mean matches accepted by AB were good matches to GH. This is a recognized characteristic of the matching performance of a dichromat.

GH noticed the same fading of colors due to retinal fatigue that the other deviates did and commented on the richness of the red colored sample, but he showed a sensitiveness to luminance and to changes in chromaticity across the long axes of his limens which apparently aided him and gave him confidence in his decisions. He did not complain of eye or physical discom-

[13] F. H. G. Pitt, Medical Research Council, *Report of the Committee on the Physiology of Vision*, XIV; Special Report Series, No. 200 (London, 1935); quoted by Judd.[1]

fort and was interested and willing to carry on the testing when needed.

GENERAL CONSIDERATION OF RESULTS

The observers responded differently to the well known Ishihara and American Optical Company pseudo-isochromatic plates and to the Glenn Colorule. Four of the observers read the plates as people with normal color vision according to the authors of the tests, and the remaining three observers read the plates with varying degrees of sensitivity to or perception of the confusion patterns.

Consistent with these data from the color tests and with the known everyday color vision of the observers were the measurements by the instrument.

The location, size, shape, and orientation of the limens are characteristic of the observer.

In two series of matches separated by several days, the limens for each observer remained much the same in location, shape, and orientation but decreased in size in the second series, as if experience in working the instrument and studying the colors made it possible for the observer to discriminate between smaller differences in color. Such changes might be the result of differences in sensitivity of the observer on different days but, with the second series of matches consistently resulting in smaller limens, it seems more logical to attribute the change to experience, learning, and higher standards of discrimination rather than to improved sensitivity.

Variations in *location* of the limens from one observer to another were noticeable. The limens of some observers included the point of chromaticity of the corresponding sample as seen by the I.C.I. Standard Observer. The limens of the other observers did not. The limens of the normal observers tended to be nearer this point than the limens of those who showed deviation from normal color vision. The fact that the limens of the normal observers did not include the point of color for the I.C.I. Standard Observer for some samples may mean that the instrument does not have absolute accuracy as explained in an earlier section, or that the color vision of the "normal" observers did not happen to coincide with that of the Standard Observer of the I.C.I. system.

With the exception of the results for the yellow-green sample, there is a tendency for the limens of the observers to fall at longer wavelengths and lesser purity than the point representing the sample as seen by the I.C.I. Standard Observer. Some known variables in the system would point toward this systematic difference (e.g., the large angle of incidence of the beam illuminating *R.S.*, Fig. 3), but it is recognized that a number of other variables are involved, the control of which would be more laborious than this study warrants. Even with this inaccuracy relative to the standard I.C.I. system, the results indicate that the errors in comparative work are small or relatively constant and, therefore, a high enough degree of precision is maintained to be useful in this study.

Variations in the *size* of the limens were significantly different among the observers and indicated the variation in the sensitivity to or the perception of differences of chromaticity or both. As suggested above, these differences in the size of the limens may have been caused in part by differences in experience and standards of discrimination among individuals. But even after generous allowances for these causes, distinguishable differences were evident among the individual observers which were logically due to differences in sensitivity to color stimuli.

A match for one observer is not a match for another observer unless their limens overlap at the point describing the chromaticity of that match. When the areas of two observers overlap, those observers accept as matches those settings which occur in the overlapping portion of their limens.

The size of the limens of the more normal observers (e.g., AB, Fig. 7) tended to be smaller than those of the deviates from normal color vision (e.g., GH, Fig. 13).

Variations in the *shape* of the limens of the individual observers were noticeable and were probably significant, especially in the extreme cases. For example, those of GH (Fig. 13) were very long; those for FG were wide; those for BC (second series) were narrow.

Variations in the *orientation* or slope of the axes of the limens were significant among the limens of the different samples for the same individual and among the limens of the different

individual observers for the corresponding samples. One of the most interesting examples was that of GH (Fig. 13). According to Pitt,[13] this is the characteristic orientation for the deuteranope. In comparison with the great length of his limens, the width was narrow. In contrast to this type of orientation are those limens of AB (Fig. 7) and BC (Fig. 8) in which the long axes tended to lie almost at right angles to those of GH.

The preceding data and discussion of the variations in the location, size, shape, and orientation of the chromaticity limens of the observers show how individuals vary among themselves and suggest further discussion of the relation of these differing limens to the corresponding sensitivity to color, the classification of observers, and theoretical considerations of color vision. However, such discussion will be postponed until more data are available.

If observer GH is a true deuteranope—all results obtained thus far show that he is—in agreement with the findings of Pitt,[13] he confuses colors whose chromaticities lie approximately on straight lines. It is of interest to note, however, that his limens do not, in all cases, extend over the maximum possible distance from one side of the spectrum locus to the other. In the case of the blue color, the limen extends the maximum possible distance—that is, from the boundary of the zone of purples (which cannot be entered with the present apparatus) to the spectrum locus. However, his limens for the blue-green and yellow-green colors extend over most *but not all* of the possible intervals. He does not confuse *all* real colors whose chromaticities lie along the system of straight lines. This is in agreement with the findings of MacAdam,[14] who has reported that none of the dichromats tested by him confused *all* colors along any locus crossing the diagram. Care was taken to see that the ends of GH's limens falling inside the chromaticity diagram were not spuriously the result of an apparatus limitation; when matches were made at or near such ends of limens, the physical adjustments of luminance, purity, and wave-length were each well within its respective range.

[14] David L. MacAdam, Paper presented before meeting of the Optical Society of America, March 6, 1943.

The orientation of the limens for AB and BC were similar to those of the ellipses reported by MacAdam.[8] In comparing the results of the present work with those published by MacAdam, consideration should be given to at least three essential differences between the two investigations: (1) As discussed earlier in this report, the present work involved variation of luminance, thus causing the limens to approximate the *projections* on the chromaticity plane of the three-dimensional limens. In MacAdam's work, the design of the experiment was such as to hold luminance constant during the matching and, hence, the ellipses reported by him are *cross sections* of the corresponding three-dimensional figures. The elongation of most of the limens observed for AB and BC is taken as evidence for the view that the three-dimensional limens are not oriented with their long axes vertical (parallel with the axis of luminance), but are inclined. It is obvious that the projections of inclined limens on the chromaticity plane will be as long or longer—possibly much longer—than the major axis of a cross section of a limen. (2) In the case of the results reported by MacAdam, the distance from the center of an ellipse to the ellipse is the *standard deviation* of a series of values observed in making color matches. In the present work, a definite attempt was made to determine the *thresholds* of perception of color difference. The exact ratio of a threshold differential to the standard deviation is not known but, without doubt, the threshold differential is a few times larger than the standard deviation. (3) The circular area of the matching fields in the present work was surrounded by a field of low (but not imperceptible) luminance, whereas the surrounding field in MacAdam's apparatus was held at an appreciable level (but less than that of the area of match), which is known to result in improved sensitivity. All three of the foregoing differences between the conditions obtaining in this and in MacAdam's work cause the limens appearing on the chromaticity diagrams presented in this report to be much larger than the ellipses representing standard deviations at constant luminance in similar areas of the diagrams published by MacAdam.

The interesting "tails" or appendages appearing on CD's limens for the blue-green, yellow-

green, and orange colors are possibly due to the acceptance of color differences as "matches" which were not quite true matches for that observer. When another normal observer (BC) deliberately accepted near but noticeably different matches at the low-purity end of his limen for the yellow-green color, a "tail," curving away from the major axis, was obtained (see Fig. 8).

There is abundant evidence in the data reported herein to the effect that the three-dimensional limens do not even roughly approximate such simple geometrical shapes as ellipsoids.

PSYCHOLOGICAL PRECAUTIONS

As the measurement of color vision includes physics, physiology, and psychology, precautions must include all three. Mention is made in the following paragraphs of some of these psychological precautions, and they, in turn, together with the physical arrangements of the testing instrument, include some of the physiological precautions. Most of the psychological conditions suggested by Dimmick[2] for reliable observations in testing have been met.

The response of the observer was simple. After adjusting only the brightness of the illuminant on the colored sample, the observer indicated whether the two halves of the field match in color.

The size of the image (3.2 degrees) is constant and sufficiently large for reasonably good color discrimination.

The whole visual field surrounding the judgment area was constant, because the observations were made in a dark room under standard conditions wherein the visual conditions remained the same for all observers.

The length of the lineal contact between the areas juxtaposed and the width of the line of demarcation remained constant by construction of the instrument.

The spatial relation was the up-down arrangement recommended by Dimmick[2] (page 313), but it was obviously not possible to duplicate all judgments using both directions by interchanging the halves of the judgment area.

Binocular vision was employed for all observers. Six of the seven observers normally wear glasses and wore them while making their color judgments. None of the lenses were tinted.

Time errors were undoubtedly present because matches for all samples were not studied on the same day; some observers spent much more time in making judgments than others; and the color-weak observers showed more signs of chromatic fatigue than the normal observers. These differences might have been controlled by giving more attention to the time factor in presentation of the stimuli. A haphazard order was followed in presenting the stimuli to all observers, and the observers were permitted to rest their eyes and check their judgments. In practical situations, these time and order factors are not controlled.

The intensity of the visual stimulation of the judgment area remained within the limits of fair discrimination and averaged about 2 millilamberts, as described in a preceding section. This corresponds, in ordinary inspection, to a level of illumination of about 5 footcandles on the colored papers.

The number of judgments required depends on a number of variables. With the great flexibility of the instrument, it was possible to get numerous judgments rendered in the simplest, and, therefore, the most stable form by requiring the observer to respond "same" or "different" at any point on the color diagram and to repeat settings until limen points were stable without the observer knowing what his results were. This procedure resembles the method used to further complicate the judgment which was found by Dimmick[2] (page 314) to be advantageous.

The instruction of the observers was explicit and standard and included a demonstration of the operation of the controls of the instrument with the corresponding changes in the appearance of the judgment field, and some practice.

In this connection reference is made to the psychological difficulty experienced by observer FG in accepting matches after he understood the nature of the testing procedure and recognized his own limitations in color vision.

The spirit of competition and unwillingness to accept the truth regarding deficient color vision appeared to influence the attitude of observers, even though the supervisor attempted to create the atmosphere that the testing was for diagnostic and experimental purposes only.

The intelligence of the observers could not have been a limiting or discriminating factor in the present work.

Other psychological and physiological factors which appear to enter this field of testing color vision are the physical condition and mental health of the subjects. For example, a change in the location of a limen after several days would tend to indicate a change in the physiological equipment of the subject. Likewise the temperament of a subject or a temporary emotional condition or attitude may influence his decisiveness or patience with corresponding results in his record of judgments.

APPRAISAL OF THE INSTRUMENT

The instrument may be evaluated in at least a qualitative manner by considering the influences of the salient apparatus variables and the results obtained. The more important variables include: (1) *Wave-length calibration.* As discussed in an earlier section, the wave-length scale is checked immediately before and after a series of tests with the help of a stenochromatic filter of effective wave-length 577.8 mμ. The uncertainty in this check is of the order of 0.002 inch on the wave-length micrometer, and the drift from day to day is ordinarily less than this figure; the wave-length uncertainty corresponding to 0.002 inch is roughly 0.5 mμ. In view of the previously mentioned correction applied to the micrometer reading, it is felt that 0.5 mμ represents the maximum possible deviation from constancy of the wave-length scale. This uncertainty would produce a shift on the chromaticity diagram of negligible value where deviates are concerned, and a scarcely significant shift in the case of normals only in the part of the diagram in the neighborhood of the blue color; in no case would a distortion of a limen determined in a single sitting be of any significance. (2) *Purity control.* By construction of the instrument, the opening of the white-light slit is directly equal to the change in the purity micrometer setting and, hence, the precision of the *mechanical* control is very high. As previously discussed, changes in zero settings are allowed for; no shifts on the diagram are attributable to instrumental error in this mechanical control. The purity, however, is also influenced by variations in both (a) the relative spectral energy distribution of the light and (b) the distribution of energy in a vertical direction in the plane of the slit mechanism. These distributions of energy are affected primarily by aging of the projection lamp (L_1, Fig. 2), which should become detectable when the white-light matches are made. The smaller lamp (L_2) is of a type made to close tolerances. The latter lamp is changed frequently, so that there is no danger of its aging in a manner paralleling that of the projection lamp. It is estimated that precision suffers from variations in the above-mentioned energy distributions only over long periods of time, and then probably to an almost negligible extent. Over the short periods required to determine the series of limens for a given subject, and to check them against an observer of normal color vision, it is believed that the source of error under discussion has negligible effect. (3) *White-light match.* Probably the most significant source of variation in the method is the adjustment of the voltage on lamp L_2 to obtain the white-light match (as described in an earlier section). When the color temperature of this lamp is varied, the chromaticity of the "white" light to be matched varies along a slightly curved line closely paralleling the Planckian locus. The maximum uncertainty in the chromaticity of the light illuminating *R.S.* (Fig. 3) introduced by the uncertainty in making the white-light match can be estimated from the observed extremes in voltage of the lamp L_2 during the matching. On translating from voltage range to color temperature range, it is found by reference to the Planckian locus on the I.C.I. chromaticity diagram that the range of chromaticity uncertainty is of the order of ± 0.0015 unit. The corresponding range of shift of the limens is certainly negligible where subjects of abnormal color vision are under study, and is small when compared with the relative displacements of the limens of the normals which have thus far been investigated. However, in a critical study of the day-to-day variability of a given normal, or in a close comparison of very similar, sensitive normals, this source of variation would not be unimportant unless the white-light match was the average of a quite large number of individual settings, at frequent intervals.

The results obtained with the instrument cor-

relate with the data obtained for the deviates with the Ishihara, American Optical Company, and Glenn Colorule tests and with the commonplace observations of the color matching performances of the subjects. Such correlations alone do not, of course, justify the addition of a new method for color-blindness testing to an already long list; however, a research method must meet a number of expectations, the most rudimentary of which is correlation with practical experience. Probably the most valuable feature of the method employed in this work is that one obtains, for each subject, a chart of the subject's limens in very fundamental terms. The limens on the chromaticity diagram are found to be characteristic of the observer and may be used in a basic manner to explain and understand better the color-matching performance of the observer. Many illustrations could be made, a few of which are: (a) the chart shows the areas in which the subject will be relatively sensitive or insensitive, (b) the locations of the limens enable one at once to ascertain the areas (if any) in which the subject will obtain matches which are distinctly not matches to an arbitrarily selected observer of normal color vision and, conversely, in which matches to the normal may not be matches to the subject, and (c) the set of characteristic limens should aid in classifying the color weak and color blind. In contrast, existing simple tests do not furnish fundamental data which may be used to explain or forecast the color matching performance of an individual.

Good evidence of reality of some of the observed shifts in the limens for specific subjects is found in the observation that two series of determinations for the same subject may differ only in that one or two, *not all*, of the limens are appreciably shifted (on comparing one set with the other set). If such shifts had instrumental origin, it would be difficult to explain why all of the limens were not shifted.

Another feature of the instrument and method is its flexibility. The matching system gives complete quantitative control, over most of the chromaticity diagram (with some limitations in luminance), through the simple turning of two micrometers. This makes it possible to produce light of any desired chromaticity in a few seconds.

It appears justifiable to conclude that the I.P.C. instrument and method described herein possess the precision and validity required to differentiate between the color vision of different individuals.

It is felt that the method would be improved by provision for a field surrounding the aperture, of variable luminance and color.

SOME PRACTICAL IMPLICATIONS

The following example will illustrate the connection between the chromaticity limens for several persons and the kind of disagreements which often arise between persons engaged in color matching. Let us suppose that three persons are attempting to match the color of a submitted sample (*S*). If tested by such ordinary methods as that of Ishihara, it might be found that Messrs. Smith and Jones were "normal," but that Mr. Anomal was "color weak"; nevertheless, all three men have matched colors for years with some degree of success. Occasionally there had been arguments similar to the one of the moment. Smith, Jones, and Anomal prefer, respectively, three different matching attempts, which we will designate as *A*, *B*, and *C*. Smith can see very noticeable differences between the matches of Jones and Anomal and the submitted sample; Jones can see very noticeable differences between the matches of Smith and Anomal and the sample; Anomal finds himself in disagreement with Smith, but will accept the match of Jones. Although the color vision of Smith might be most closely similar to that of the average person of normal vision, the opinion is two-to-one in favor of match *B*, and probably the best match, *A*, is not used.

The hypothetical chromaticity limens for Smith, Jones, and Anomal are shown in Fig. 14, and the chromaticities of the submitted sample and of the three matches are designated by the letters *S*, *A*, *B*, and *C*.* It will be seen that the

* The limen for a given observer, and for a given color to be matched, will depend upon the spectral compositions of the matching color and the color to be matched. For example, the location, size, and shape of a limen determined with a combination of white light and spectrum light will be substantially different from those of a limen determined with a matching light of spectral composition differing in small steps from that of the color to be matched. The former *may* bound an area of match which *does not include* the chromaticity of the color to be matched, whereas the latter, no matter how extensive, will always

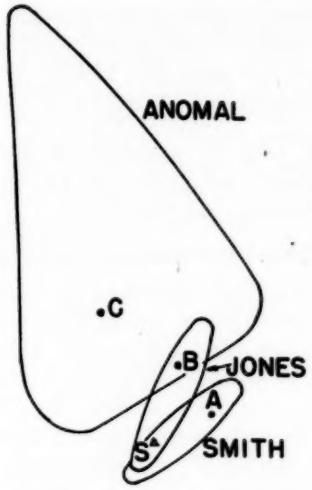

FIG. 14. Illustrative hypothetical limens.

limens are not of the same size and orientation, but that the areas of match overlap to some extent. The extensive area for Anomal shows that his color vision is not as sensitive as that of a person of normal visual response and, more important from a practical viewpoint, the displacement of the whole area defined by his limen is indicative of the probability that his color matches will not be matches for the normal and, *vice versa*, matches for the normal will not be matches for him. (An individual having this sort of color vision is known as an anomalous trichromat.)

Smith's match, represented by the point *A*, falls outside the area bounded by Jones' limen. Since the limen is, by definition, the threshold of noticeable difference, Smith's match will not appear to Jones as a satisfactory match with the submitted sample. Since the point *B* for Jones' match falls outside Smith's limen, Jones' match will not appear to Smith as a satisfactory match with the submitted sample. Similarly, Anomal's match will be unsatisfactory to Jones and Smith —decidedly more unsatisfactory to Smith than to Jones, and Smith's match will be noticeably imperfect, although not decidedly so, to Anomal. However, Jones' match will appear satisfactory to Anomal, because the point *B* happens to fall within Anomal's area of match. Smith and Jones, the two persons of normal color vision, would agree with each other only when the matches have chromaticities falling in the overlapping area of their two limens. In the case pictured, Smith and Anomal would never come to agreement. If, however, the colorants employed were very similar to those present in the submitted sample, the limens of the three persons would tend to center upon each other, and all three would find themselves in agreement for many matching attempts (see the footnote to this section).

During the course of the work reported herein, the supervisor noted on many occasions quite decided color differences between the fields matched by subjects who were classed as "normal" by the Ishihara and American Optical Company tests. On these occasions the limens for the subjects would be displaced from the corresponding limens for the supervisor (AB) by appreciable distances on the chromaticity diagram. The reader will find a number of instances of this on comparing Figs. 7, 8, 9, and 10.

Such an example as just given prompts the question of the usefulness of this instrument as a testing device as well as a research instrument. Although it possesses a number of the disadvantages of a laboratory instrument and requires time to use, it does possess other advantages mentioned above which offset the disadvantages to the extent that it could be used profitably in the selection or diagnosis of color matchers or color technicians who are responsible for color choices which involve considerable expense in a manufacturing process and where mistakes in judgment cause expensive waste, or in the selection and diagnosis of personnel in transportation, military services, aviation, etc., where mistakes in color judgment under adverse conditions might mean loss of life as well as property.

When the usefulness of this instrument with

encompass the chromaticity of the color to be matched. The different spectral compositions of light reflected from colored papers or other bodies containing different colorants will cause the corresponding limens to be displaced on the chromaticity diagram in much the same manner as in the present work, where displacement is attributed to the fact that the matching light is composed of white light and spectrum light as opposed to the more heterogeneous nature of the light to be matched. For the sake of the present discussion, it is assumed that the spectral composition of the matches *A*, *B*, and *C* is similar to that of the matching light employed in this study; actually, of course, the spectral composition of light reflected from colored bodies is more heterogeneous.

some refinements in research is considered, several current questions come to mind.

Several industries are concerned with the problems of color specifications and color tolerances. A solution of these problems requires the further fundamental study of the limens describing the "solid of match" or the three-dimensional description of color vision.

The physiological aspects of color vision have been mentioned above. Additional information is needed on color vision variations which are innate, acquired, progressive, related to metabolic and toxic conditions, vitamin feeding, retinal correlates, surgery, adaptation, fatigue, etc. The apparatus and method described herein might furnish fundamental data on these effects.

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Changes in Hue, Lightness, and Saturation of Surface Colors in Passing from Daylight to Incandescent-Lamp Light

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198 Munsell samples, representing 13 hues at various value levels and chromas, including 9 neutrals, were viewed separately on black, gray, and white backgrounds under Illuminant *A* and matched by samples viewed in Illuminant *C* by 7 observers, making a total of over 4000 matchings. With these samples, hues containing red and blue become redder in Illuminant *A* as compared with Illuminant *C*, while hues from Munsell purple-blue to yellow-red shift toward reddish-yellow. Two invariant hues emerge, one between 10R and 5YR with all backgrounds, the other between 5PB and 10B with white background, and between 5PB and 10B with gray and black backgrounds. Greatest shifts occur with black background, least with white. General trends are qualitatively in accord with the Helson-Judd principles of color conversion and in the direction to be expected from the I.C.I. specifications of the samples in the two illuminants. The changes in hue and chroma and the invariant hues as they appear in the I.C.I. plots are not far different from those observed, but the computed changes are much greater than the observed because the I.C.I. computations are based on a completely non-reflecting background and observers screened from the illuminants. Further theoretical treatment of the data seems warranted.

I. INTRODUCTION

THE problem of matching surface colors in different illuminants is as old as the problem of color constancy. Katz[1] matched colors of surfaces that were differently illuminated in his experiments on brightness constancy. In these experiments, however, the illumination was constant in quality, differing only in amount. His attempts, as well as those of others to match colors in qualitatively different illuminants, were not satisfactory and it has been generally believed that it cannot be done successfully. If the illuminants do not differ too much in quality, such as Illuminants *C* and *A*, observers can match samples satisfactorily with respect to hue, lightness, and saturation and the results are found to be statistically reliable.

The general directions of change to be expected from the use of qualitatively different illuminants are those formulated by Helson[2,3] and Judd[4] in

* Now with Eastman Kodak Company, Rochester, New York.

[1] D. Katz, *The World of Colour* (Kegan Paul, Trench, Trubner and Company, Ltd., London, 1935), translated by R. B. MacLeod and C. W. Fox.

[2] H. Helson, "Fundamental problems in color vision. I. The principle governing changes in hue, saturation, and lightness of non-selective samples in chromatic illumination," J. Exper. Psychol. **23**, 439 (1938).

[3] H. Helson and V. B. Jeffers, "Hue, lightness and saturation of selective samples in chromatic illumination," J. Exper. Psychol. **26**, 1 (1940).

[4] D. B. Judd, "Hue, saturation, and lightness of surface colors with chromatic illumination," J. Research Nat. Bur. Stand. **24**, 294 (1940), RP1285; also J. Opt. Soc. Am. **33**, 355 (1943).

the principle of color conversion. According to this principle hue, lightness, and saturation depend not only upon spectral reflectance but also upon the relation of sample reflectance to adaptation level of the eye. Since reflectance of background determines adaptation level to a great extent, this feature of the viewing situation must be taken into account along with the spectral reflectances of the samples being matched in the two illuminants.

The experiment reported here was performed, at the suggestion of Miss Dorothy Nickerson, to establish the relationship of the colors perceived to belong to the samples of the *Munsell Book of Color*[5] under standard Illuminant *A* (representative of incandescent-lamp light), to those perceived to belong to the same samples under standard Illuminant *C* (representative of average daylight).[6] The experiment was set up in such a way as to test the validity of the color conversion principle in this particular situation. The interplay of all factors in the field that affect the three dimensions (hue, lightness, saturation) of the color perception, must be shown to be operating according to this principle in order to substantiate it.

II. PROCEDURE

The observers in this experiment were five undergraduates and two graduate students at Bryn Mawr College. Four undergraduates had never observed in color experiments before, but the graduate students and one undergraduate had observed in color experiments previously, and were observing in another experiment during the period covered by this study.

The observations were made in a dark room at two booths built side by side on a table. Illuminant *A* was supplied in the booth to the left of the observer by a Macbeth lamp containing one 200-watt bulb used without filter. An approximation to Illuminant *C* was supplied in the booth to the right by a Macbeth lamp containing four 200-watt bulbs with suitable filters of Corning Daylight glass. The illuminants were calibrated by means of a standard lamp having a color temperature of 2848°K and approximately 6700°K when viewed through a suitable blue filter, both lamp and filter having been obtained from the National Bureau of Standards. The two illuminants were found to approximate Illuminants *A* and *C* so closely at average line voltage that no further control of the current was deemed necessary for the purposes of the experiment, especially in view of the fact that any changes would affect both illuminants. No certain variations in the reports were introduced from this source so far as could be ascertained. During each step of the experiment the entire booths were covered with a different cardboard—white (N 9.5/), gray (N 5.75/), or black (N 1.25/). The illuminance of the two booths was closely the same, 50 footcandles. The color chips were placed on a small sloping stand, which had an angle of approximately 17°. This, too, was covered with cardboard like that lining the booths. The cards on which the chips were pasted were covered with shields of the same colors as the background, these shields allowing only one chip to be seen at a time.

The color chips selected for matching were limited to the 10 major hues of the *Munsell Book of Color*.[5] For the sake of convenience, these color chips which were placed in the booth under Illuminant *A*, were pasted on 3-×5-inch filing cards. The match was found by reference to a series of 40 Munsell constant-hue charts viewed in the second booth under artificial daylight (close color match for Illuminant *C*). The shields were used in both booths in order to expose only a single chip at a time. The chips chosen to be matched varied systematically in value and chroma. In each hue the 2/2, 5/2, and 8/2 chips were used, or, in other words, the chips of least Munsell chroma at a low, a medium, and a high Munsell value. Viewed in daylight these chips yield color perceptions of low saturation at a low, a medium, and high lightness. Chroma /6 was selected at 5/ value, and at the highest and lowest values at which it was represented in the Munsell papers. This procedure was followed with chroma /8, and so on; the highest chroma was always used at each value. Supplementary chips were added after preliminary experiments on the gray

[5] *Munsell Book of Color*, Pocket Edition, 40 hues (Munsell Color Company, Baltimore, Maryland, 1942).

[6] K. L. Kelly, K. S. Gibson, and D. Nickerson, "Tristimulus specification of the *Munsell Book of Color* from spectrophotometric measurements," J. Research Nat. Bur. Stand. **31**, 55 (1943), RP1549; also J. Opt. Soc. Am. **33**, 355 (1943).

background had shown gaps, reversals, or other critical points. Matches for these chips are included in all results presented. In addition to chips added to those of the ten major hues three other Munsell hues were used. These additional hues were necessitated by a search for chips whose hue would be invariant with change from incandescent-lamp light to daylight; they were 10 R, 10 B, and 10 PB. Nine neutral chips (N 1/ to N 9/) were also included.

The gray background was used first, followed by the white, then the black. The chips added after the observations on the gray background were presented together as a group in two consecutive observation sessions. During the sessions with the black background the added chips and hues were presented with the original chips. Hues were presented in random order to the seven observers. Value and chroma, within each hue, were randomized also.

The observers, before the experiment began, were shown the old Munsell Atlas and the present *Munsell Book of Color*. The Munsell system[7] was explained to them with emphasis on the fact of the arrangement of hues in a circle, thereby enabling the observers to obtain a notation for any hue by going one way or the other around the circle. The four undergraduate observers arranged loose chips on several Munsell Student Series charts in order to familiarize themselves with the Munsell attributes, value and chroma, which correlate closely under ordinary conditions of viewing with lightness and saturation. They then made matches under one light, an ordinary desk lamp. This preliminary training was unnecessary with the other three observers. Matches then were made in the experimental situation with the gray background. It soon became apparent that the four lesser-trained observers were not yet sure of what they were doing or should do. Experimental observations were stopped and the observers again matched chips under a single illuminant. The reasons for difficulties were discussed and the training continued until each observer could make satisfactory matches for value, or chroma, or for all three dimensions together. Observations on the gray background were repeated, followed by observations on the white and the black backgrounds, and it is this series of observations that is reported in this paper.

In making a match, the observer looked at a sample in the *A* booth, then found a match for this color in the *C* booth by moving the shield over one of the 40 constant-hue charts so as to expose chips of different values and chromas. If the hue was not correct the observer asked for "more blue," "less green," "more red" or whatever was required. To find the match for the neutral chips, the observer reported the hue under Illuminant *A*, and charts from this region of the hue circle were supplied for finding a match. Interpolations and extrapolations were permitted in all three dimensions, hue, value, and chroma, for any match.

The results were computed by finding means for hue, value, and chroma for the seven matches made for each chip. To find hue-means a scale equivalent to the 100-point Munsell hue scale was used, with precautions near the zero point on the scale to keep from getting a spurious indication of the complementary hue.

III. RESULTS

Table I presents the average matches made by the seven observers for each of the chips used in the experiment. For each match the standard deviation of the mean hue is also given. Figure 1 shows these average matches plotted on the Munsell constant-value diagrams at the 2/, 5/, and 8/ value levels. Results for each of the three backgrounds are shown on the left half of the chart by open circles for the white background, triangles for gray, and solid circles for black. Each of these points is connected by a straight line to the point representing the Munsell hue and chroma of the chip that was placed in Illuminant *A*. The three vectors starting from this point therefore indicate the change in color perception of the chip, caused by moving it from the space illuminated by artificial daylight into that illuminated by incandescent-lamp light.

No systematic investigation of the ability of a single observer to duplicate her estimates on a second trial was made; but one observer repeated observations on the chips of two hues through an error on the part of the experimenter. Table II contains the results of the two sets of matches.

[7] F. G. Cooper, *Munsell Manual of Color* (Munsell Color Company, Baltimore, Maryland, 1941).

TABLE I. Average match to Munsell standards in Illuminant *C* of a selected series of Munsell color samples viewed in Illuminant *A* on each of three backgrounds.

Sample in Illuminant *A*		Match in Illuminant *C* White background			Gray background			Black background		
		H	σ	*V/C*	*H*	σ	*V/C*	*H*	σ	*V/C*
5.0R	2/2	7.9R	2.1	2.1/2.6	7.2R	3.7	2.2/3.1	10.0R	2.4	2.8/3.6
	2/6	6.9R	2.8	2.4/6.0	6.7R	2.4	2.5/7.5	7.9R	0.6	3.1/6.4
	3/6	7.1R	1.0	3.3/8.6	6.1R	1.2	3.5/9.3	7.3R	0.9	3.1/6.4
	3/10	6.6R	1.1	3.2/10.7	6.0R	1.2	3.4/10.9	7.2R	0.9	3.7/11.4
	4/6	7.6R	0.9	4.0/9.0	7.3R	0.9	4.4/8.9	7.7R	0.2	4.8/9.4
	4/14	5.7R	0.6	4.6/15.0	5.5R	0.3	4.9/15.1	6.6R	0.8	4.6/14.3
	5/2	9.4R	2.9	4.9/2.3	7.7R	2.7	5.4/2.7	0.1YR	2.0	5.5/3.9
	5/6	7.1R	1.1	5.0/7.9	6.6R	1.1	5.7/9.1	7.7R	0.3	5.5/9.7
	5/10	5.8R	0.5	5.1/11.9	5.9R	1.2	5.7/13.1	7.5R	0.2	5.4/12.9
	6/2	8.0R	3.5	6.0/2.5	7.1R	0.5	6.2/2.9	1.8YR	4.3	6.9/3.4
	6/6	8.1R	0.9	6.1/8.1	7.3R	1.0	6.8/9.5	8.8R	0.9	6.6/9.7
	6/10	6.5R	0.9	6.2/12.4	6.5R	1.1	6.5/13.0	7.7R	0.5	6.1/13.3
	7/2	7.9R	3.7	7.0/2.6	7.8R	0.4	7.4/3.8	1.1YR	3.5	7.6/4.5
	7/6	8.1R	1.3	7.1/7.1	7.7R	1.9	7.5/7.5	9.4R	1.6	7.1/8.6
	8/2	6.7R	1.3	8.0/1.9	7.9R	4.5	8.4/2.8	5.9YR	2.8	7.9/4.3
10.0R	2/2	7.5R	1.8	2.3/4.0	0.3YR	2.6	4.6/4.7	1.1YR	1.2	2.8/4.7
	3/2	0.6YR	1.1	2.7/3.0	1.6YR	1.1	3.1/4.0	1.8YR	1.1	3.5/4.0
	3/6	9.4R	1.6	3.2/7.3	0.4YR	0.8	3.3/8.1	0.7YR	0.8	3.8/7.7
	4/2	1.1YR	2.0	4.0/3.0	1.5YR	1.7	4.2/3.5	2.3YR	1.8	4.8/4.1
	4/6	9.8R	1.2	4.2/8.1	9.8R	1.4	4.6/9.6	0.6YR	0.8	4.7/9.9
	4/10	9.0R	1.3	4.4/11.9	0.1YR	1.4	4.8/12.0	1.6YR	2.5	4.9/12.3
	5/2	2.1YR	1.7	5.0/2.9	1.2YR	1.9	5.5/2.6	2.5YR	1.7	5.6/4.3
	5/6	1.6YR	4.7	5.0/9.1	0.5YR	0.9	5.5/9.2	1.2YR	1.1	5.4/10.5
	5/10	9.3R	1.3	5.0/12.0	9.2R	1.9	5.6/12.3	0.6YR	0.8	5.5/13.3
	6/2	1.0YR	2.1	6.1/2.9	1.8YR	1.7	6.4/3.2	4.1YR	2.3	6.7/4.4
	6/6	1.1YR	1.4	6.2/8.4	0.8YR	1.1	6.4/8.6	1.4YR	1.0	6.7/8.9
	6/10	0.4YR	1.0	6.3/13.4	0.1YR	1.2	6.3/13.4	1.0YR	1.0	6.2/13.3
	7/2	0.7YR	1.9	7.2/3.0	1.4YR	1.8	7.6/4.3	5.0YR	1.8	7.6/5.5
	7/6	0.9YR	2.4	7.3/7.8	0.4YR	0.9	7.3/9.4	1.5YR	1.3	7.2/8.7
	8/2	0.4YR	1.6	8.1/2.6	0.7YR	1.7	8.3/3.6	6.2YR	2.3	8.1/5.1
5.0YR	2/2	2.8YR	3.4	2.4/2.4	4.3YR	1.8	2.3/2.1	3.9YR	0.9	3.1/2.9
	3/2	2.3YR	2.1	2.8/3.1	4.0YR	2.1	3.0/3.4	4.5YR	0.1	3.4/4.3
	4/2	5.6YR	1.8	3.9/2.9	5.0YR	1.1	4.2/3.3	5.4YR	0.9	4.6/3.9
	4/6	3.7YR	1.1	4.1/7.5	4.1YR	1.0	4.5/8.0	3.7YR	0.8	4.7/8.7
	5/2	6.4YR	1.7	4.9/2.2	5.5YR	1.6	5.3/2.4	5.2YR	1.0	5.6/3.8
	5/6	4.5YR	1.1	5.0/7.1	4.0YR	0.9	5.3/7.6	4.1YR	0.8	5.4/8.5
	5/10	4.1YR	1.1	5.2/10.3	3.1YR	1.2	5.5/11.3	3.5YR	1.1	5.5/10.0
	6/2	7.1YR	0.8	5.9/2.6	4.2YR	0.8	6.3/2.7	7.7YR	1.9	6.6/3.6
	6/6	4.2YR	0.8	6.0/7.4	4.1YR	0.8	6.4/8.1	4.4YR	0.6	6.7/9.0
	6/10	3.9YR	1.2	6.3/11.6	3.1YR	0.6	6.4/11.4	3.5YR	1.0	6.4/12.4
	6/12	3.2YR	0.6	6.3/12.0	2.7YR	0.9	6.7/11.7	3.0YR	0.8	6.6/12.4
	7/2	5.0YR	2.5	6.9/2.4	3.9YR	1.6	7.3/3.4	6.3YR	1.1	7.5/4.4
	7/6	4.5YR	1.4	7.0/7.6	2.6YR	0.2	7.3/8.5	3.6YR	1.1	7.3/8.7
	7/8	3.8YR	0.9	7.2/10.1	2.7YR	0.5	7.3/9.8	3.8YR	1.1	7.3/10.9
	7/10	3.7YR	1.2	7.2/11.8	2.6YR	1.1	7.3/10.9	3.7YR	0.9	7.0/12.6
	8/2	4.9YR	1.1	8.0/2.0	2.9YR	1.5	8.2/2.5	6.3YR	2.2	8.2/4.6
5.0Y	2/2	5.9Y	1.1	2.3/2.1	4.4Y	2.1	2.1/1.9	3.2Y	1.1	3.2/2.4
	5/2	4.7Y	1.2	5.0/2.6	4.3Y	0.9	4.9/2.3	3.0Y	1.8	5.5/3.4
	5/6	4.3Y	1.4	5.1/5.1	4.1Y	1.2	5.1/6.1	3.5Y	1.4	5.6/6.7
	6/2	5.1Y	0.7	5.9/2.4	3.2Y	2.2	6.1/2.4	2.1Y	1.9	6.6/3.7
	7/2	4.9Y	0.4	6.8/2.4	1.9Y	2.4	7.4/2.6	2.1Y	1.9	7.5/3.4
	7/6	4.4Y	1.0	6.8/5.9	2.6Y	1.2	7.3/7.6	1.8Y	1.7	7.3/8.2
	7/10	4.5Y	0.9	7.1/7.7	2.9Y	1.0	7.4/8.9	2.8Y	1.3	7.4/10.1
	8/2	5.4Y	0.8	7.9/2.3	2.2Y	1.6	8.4/2.3	1.6Y	1.8	8.0/4.1
	8/4	5.1Y	0.4	7.9/4.2	3.0Y	0.4	8.1/5.1	2.5Y	1.7	8.0/5.9
	8/6	4.6Y	0.7	7.9/6.1	3.8Y	1.1	8.1/6.2	1.9Y	0.7	8.0/8.2
	8/8	4.6Y	0.7	7.9/7.9	3.1Y	1.2	8.2/8.4	2.9Y	1.7	8.0/9.8
	8/10	4.5Y	0.6	8.0/9.3	3.0Y	0.8	8.1/9.6	1.9Y	0.6	8.0/10.9
	8/12	4.1Y	0.9	8.0/9.9	2.5Y	0.7	8.1/10.1	2.4Y	1.3	8.0/11.5
5.0GY	2/2	5.2GY	2.8	2.1/2.0	2.1GY	2.5	2.1/1.9	2.0GY	3.0	2.9/1.9
	3/2	3.4GY	2.5	3.1/2.2	2.3GY	2.3	3.1/2.2	1.4GY	2.7	3.7/2.3
	4/2	2.7GY	2.4	3.8/2.5	3.4GY	2.5	4.2/2.1	1.1GY	2.6	4.5/2.6
	5/2	4.3GY	1.2	4.9/2.1	3.0GY	1.8	5.1/2.2	0.2GY	2.5	5.7/2.7
	5/6	4.9GY	1.6	4.9/6.1	3.9GY	1.2	4.9/4.9	3.4GY	1.1	5.5/5.7
	5/8	4.8GY	1.5	4.8/7.4	5.0GY	1.1	5.2/5.9	3.4GY	1.0	5.5/7.2
	6/2	2.3GY	4.5	5.9/2.3	1.4GY	2.2	6.2/2.1	0.1GY	2.5	6.5/3.0

TABLE I.—*Continued.*

Sample in Illuminant *A*		White background			Match in Illuminant *C* Gray background			Black background		
		H	σ	*V/C*	*H*	σ	*V/C*	*H*	σ	*V/C*
5.0GY	7/2	3.4GY	1.7	6.9/2.3	1.6GY	2.5	7.3/2.2	9.7Y	2.9	7.7/3.3
	7/6	3.8GY	1.6	6.8/5.8	3.2GY	1.6	7.3/5.7	2.6GY	1.8	7.4/6.0
	7/8	3.9GY	1.4	6.9/7.0	3.3GY	1.4	7.3/6.5	2.2GY	1.4	7.3/7.1
	7/10	4.8GY	1.2	6.9/7.7	3.7GY	1.1	7,4/7.6	2.8GY	1.4	7.4/8.4
	8/2	3.3GY	1.6	7.8/2.0	0.9GY	1.7	8.5/2.1	8.2Y	5.6	8.2/3.1
	8/4	3.2GY	1.2	7.9/4.1	1.1GY	1.8	8.1/4.1	9.1Y	3.5	8.0/5.0
	8/6	4.6GY	0.8	7.9/5.5	2.0GY	1.9	8.1/5.5	0.4GY	2.8	8.0/6.6
	8/8	4.1GY	1.1	7.9/7.2	0.9GY	4.3	8.1/6.5	1.3GY	2.2	8.2/7.5
5.0G	2/2	6.4G	4.2	2.0/1.6	1.4G	5.1	1.9/1.8	0.5G	4.2	2.0/2.1
	3/2	6.4G	4.6	3.0/2.8	3.6G	6.7	2.8/1.7	9.8GY	3.9	3.4/1.6
	4/2	0.6G	5.0	4.0/2.7	3.9G	2.8	4.1/2.9	1.1G	2.9	4.6/2.1
	5/2	2.5G	2.9	4.7/2.1	2.5G	6.7	4.9/2.1	7.5GY	3.0	5.4/2.6
	5/4	2.6G	3.1	4.7/4.4	3.9G	2.7	4.8/3.7	0.1G	2.6	5.1/4.1
	5/6	3.2G	2.2	4.5/5.9	3.1G	1.8	4.9/5.5	0.9G	1.2	5.2/5.7
	5/8	3.8G	1.5	4.7/6.9	3.6G	1.7	4.8/7.8	2.0G	0.6	5.2/7.6
	8/2	2.6G	2.5	7.7/2.3	2.8G	2.7	8.1/2.0	7.3GY	4.3	8.0/3.5
	8/4	2.5G	2.4	7.7/3.6	3.0G	2.7	7.1/2.7	0.2G	1.9	8.2/4.1
	8/6	3.4G	2.0	7.5/5.3	2.1G	2.3	7.8/5.1	9.4GY	2.3	7.9/5.7
5.0BG	2/2	9.3G	8.8	1.9/2.2	2.7BG	4.4	1.9/1.8	0.2BG	4.8	2.3/1.5
	3/2	9.6G	6.0	2.7/1.9	1.4BG	8.9	2.9/1.5	7.4G	4.9	3.4/1.4
	3/6	3.2BG	5.8	2.8/4.5	5.0BG	1.1	3.3/4.2	0.6BG	1.9	3.5/3.9
	5/2	8.7G	5.7	4.9/2.1	0.2BG	5.0	5.1/2.0	2.0G	4.5	5.6/1.8
	5/4	10.0G	3.7	4.8/4.0	1.1BG	3.3	4.9/3.4	5.9G	2.9	5.6/3.4
	5/6	0.6BG	3.4	4.9/5.0	1.3BG	2.5	4.9/5.4	6.8G	1.9	5.6/5.0
	6/6	0.3BG	3.8	5.9/4.6	10.0G	3.1	6.0/4.1	4.8G	3.5	6.7/5.3
	8/2	9.1G	5.9	7.7/2.0	7.1G	6.2	8.4/1.5	8.2GY	8.5	8.2/2.9
5.0B	2/2	6.4BG	15.1	1.9/2.1	9.0B	4.9	1.9/1.8	0.7PB	6.9	2.5/1.4
	3/2	8.4BG	5.0	2.5/1.9	0.9B	7.5	3.0/1.6	6.8BG	5.7	4.1/1.2
	3/6	3.0B	3.5	3.0/3.9	2.9B	2.5	2.9/4.3	9.6BG	2.4	3.5/4.1
	4/2	0.9B	2.6	3.8/1.9	9.3BG	6.4	4.1/1.6	5.3BG	6.1	4.9/1.0
	4/4	2.1B	4.2	3.8/2.8	2.5B	2.4	4.0/2.7	8.2BG	2.9	4.6/1.9
	4/6	3.5B	1.8	3.5/4.7	4.5B	5.1	3.7/4.6	0.5B	2.5	4.2/3.9
	4/8	2.3B	4.0	3.7/4.6	2.5B	3.0	3.8/4.4	9.5BG	2.5	4.3/4.0
	5/2	6.4BG	8.8	4.8/1.9	0.9B	4.2	5.0/1.4	0.4BG	9.8	5.6/1.1
	5/4	0.3B	3.1	4.7/3.6	0.7B	2.4	4.9/3.1	5.2BG	3.1	5.4/2.7
	5/6	1.1B	4.0	4.5/5.0	0.9B	2.3	5.0/4.8	5.1BG	5.3	5.3/4.6
	6/6	10.0BG	2.0	5.6/4.4	9.5BG	1.8	6.0/4.1	5.1BG	1.7	6.4/4.2
	7/4	9.2BG	4.8	6.9/3.4	8.4BG	3.9	7.3/2.3	2.9BG	7.0	7.4/2.0
	7/6	8.8BG	4.8	6.6/3.6	9.0BG	4.4	8.3/3.6	0.7BG	6.6	7.3/2.9
	8/2	0.8B	7.0	7.8/1.8	1.4B	3.3	8.6/1.5	2.5G	17.6	8.2/1.9
10.0B	2/2	6.3PB	11.6	1.6/1.9	2.9PB	4.5	1.8/1.7	7.1PB	4.4	2.3/1.4
	3/2	7.5B	3.0	2.9/1.7	9.3B	1.7	3.0/0.9	4.8B	5.3	4.5/0.6
	3/6	8.5B	2.7	3.0/3.9	9.8B	1.8	2.9/3.5	6.1B	1.9	3.4/3.2
	3/8	9.1B	2.8	3.1/4.9	0.2PB	1.3	2.9/5.3	5.0B	2.4	3.1/4.3
	4/2	4.6B	6.6	3.8/1.6	8.9B	3.9	4.0/1.2	2.5B	6.3	4.8/0.8
	4/6	7.7B	2.5	3.9/3.6	8.7B	1.8	3.9/3.7	4.9B	1.7	4.5/2.5
	4/8	7.3B	3.0	3.9/4.4	7.4B	3.7	3.8/4.7	3.6B	1.8	4.4/3.8
	5/2	6.4B	2.1	5.0/1.6	8.5B	2.2	5.2/1.3	7.0BG	15.4	6.1/0.6
	5/6	6.1B	2.3	4.9/3.6	7.2B	3.1	5.0/3.5	1.8B	1.6	5.6/2.6
	6/2	5.9B	2.7	5.8/1.7	8.8B	3.0	6.4/1.3	8.0G	17.3	7.5/1.3
	7/2	7.7B	4.3	6.6/1.5	9.3B	4.0	7.5/1.4	7.1G	18.6	8.1/1.7
	7/6	6.6B	3.9	6.7/3.9	6.0B	3.5	7.1/3.3	9.8BG	6.2	7.4/2.8
	8/2	9.9B	1.1	8.0/1.9	0.5B	1.9	8.4/0.9	1.6G	21.6	8.5/1.7
5.0PB	2/2	6.8B	16.2	1.7/1.8	4.3PB	2.6	1.8/1.5	5.2PB	1.6	2.5/0.7
	2/6	4.4PB	3.5	2.1/2.9	5.7PB	1.0	1.8/2.7	5.4PB	1.1	2.4/2.5
	3/10	3.3PB	2.2	3.0/4.9	5.1PB	3.3	2.9/5.3	3.5PB	1.3	3.3/4.0
	3/12	3.8PB	2.5	3.8/6.3	5.1PB	0.1	2.8/6.1	3.9PB	1.0	3.3/4.9
	4/10	3.1PB	2.2	4.0/5.3	4.6PB	0.8	3.9/5.6	3.2PB	0.9	4.5/4.2
	5/2	2.3PB	2.1	4.8/1.4	4.2PB	1.5	4.9/1.4	5.0PB	1.3	5.9/0.4
	5/4	3.3PB	2.8	5.0/1.9	5.7PB	0.9	5.2/2.0	4.5PB	1.5	5.6/0.9
	5/6	2.8PB	2.6	4.8/2.6	5.2PB	0.2	5.2/2.6	3.9PB	2.1	5.9/1.8
	5/8	3.0PB	1.9	5.0/4.3	4.4PB	1.0	5.1/3.6	2.1PB	1.1	5.6/2.6
	5/10	1.8PB	3.1	4.9/4.7	3.9PB	1.1	5.0/4.1	2.0PB	0.9	5.9/3.9
	6/2	4.5PB	2.1	6.1/1.4	7.5PB	1.6	6.5/1.5	0.7RP	19.7	7.5/1.1
	6/6	3.6PB	2.3	5.7/2.7	5.9PB	1.4	6.2/2.4	5.8PB	2.0	7.0/1.8
	7/2	3.4PB	1.9	7.1/1.5	6.6PB	1.4	7.5/1.3	4.6RP	23.5	8.1/1.3

TABLE I.—*Continued.*

Sample in Illuminant *A*		Match in Illuminant *C* White background *H*	*σ*	*V/C*	Gray background *H*	*σ*	*V/C*	Black background *H*	*σ*	*V/C*
5.0PB	7/6	4.5PB	1.8	6.6/2.6	5.8PB	1.7	7.1/2.0	3.4PB	4.6	8.0/1.2
	8/2	4.6PB	2.7	7.8/1.5	7.9PB	5.0	8.4/1.3	0.9R	22.1	8.4/2.1
10.0PB	2/2	4.3P	7.5	2.0/1.9	7.0P	4.2	1.9/1.9	9.9P	1.7	2.4/2.0
	2/6	4.1P	7.7	2.0/2.4	7.0P	2.1	2.2/3.1	8.2P	2.4	2.5/3.0
	3/2	7.3P	9.9	2.5/2.2	7.5P	5.8	2.9/1.7	1.4RP	5.1	3.5/1.8
	3/6	4.5P	5.5	2.8/3.6	5.9P	2.4	3.1/3.6	8.9P	4.6	3.7/3.7
	3/10	4.1P	2.5	2.9/5.9	5.4P	1.6	3.0/6.4	7.0P	1.5	3.3/6.7
	4/2	4.3P	10.5	4.0/1.6	4.6P	2.5	4.2/1.6	6.4RP	10.6	4.6/1.7
	4/6	3.7P	3.8	4.2/2.6	4.4P	1.2	4.3/3.9	9.0P	3.3	4.6/3.4
	4/10	3.6P	2.7	3.8/4.4	4.4P	1.4	4.0/5.7	7.8P	2.0	4.5/4.7
	5/2	3.9P	5.6	4.8/1.8	4.5P	2.3	5.5/1.4	6.8RP	12.2	5.6/1.7
	5/6	4.3P	4.3	5.2/2.6	5.2P	2.3	5.3/3.4	9.6P	3.3	5.7/3.3
	5/10	3.9P	3.6	4.8/4.6	4.8P	1.0	5.3/5.3	7.7P	2.9	5.6/5.1
	6/2	3.9P	5.6	6.2/1.7	7.1P	3.3	6.5/1.6	9.6RP	10.0	7.1/2.0
	7/2	4.2P	5.5	6.9/1.9	10.0P	4.0	7.4/1.9	7.7R	22.6	7.8/2.7
	7/6	4.1P	3.7	6.7/3.1	5.8P	2.7	7.3/4.0	5.0RP	5.4	7.4/3.7
	8/2	4.2P	6.3	8.1/1.6	1.8RP	5.9	8.5/1.7	7.1R	11.9	8.2/3.1
5.0P	2/2	3.6RP	9.1	1.9/1.9	2.9RP	5.9	2.0/1.9	4.3RP	2.9	2.6/2.5
	2/4	6.1RP	6.1	2.0/2.4	2.9RP	6.2	2.0/3.4	4.5RP	1.7	2.5/3.7
	2/6	4.6RP	3.8	2.2/4.4	4.8RP	2.7	2.1/5.8	4.1RP	1.7	2.7/5.3
	3/6	3.8RP	4.6	3.2/4.9	2.4RP	1.6	3.0/5.5	5.9RP	5.2	3.6/5.2
	3/10	3.9RP	2.6	3.0/9.4	3.2RP	1.1	3.4/9.2	4.5RP	1.7	3.7/8.8
	4/6	2.9RP	5.1	4.1/4.7	2.5RP	3.7	4.3/5.0	4.7RP	2.7	4.4/4.7
	4/10	3.4RP	2.8	3.9/8.7	0.7RP	4.3	4.1/9.0	4.8RP	1.9	4.4/9.2
	4/12	3.6RP	2.5	3.8/10.3	4.2RP	1.8	4.2/10.9	5.0RP	1.8	4.3/10.4
	5/2	3.2RP	6.8	4.8/1.6	2.4RP	3.3	5.2/1.6	8.9RP	7.4	5.4/2.3
	5/4	2.3RP	4.8	5.0/3.2	3.0RP	3.8	5.2/3.4	5.9RP	3.0	5.4/3.9
	5/6	1.8RP	2.6	4.9/4.9	2.6RP	3.8	5.0/4.9	6.0RP	2.3	5.5/5.9
	5/8	1.6RP	3.1	4.9/5.0	2.5RP	2.3	5.2/6.6	4.3RP	2.4	5.5/7.7
	5/10	2.1RP	2.4	4.7/8.4	1.7RP	1.9	5.0/8.4	3.8RP	2.2	5.3/9.1
	6/2	3.1RP	7.4	6.1/1.8	1.4RP	4.4	6.2/2.4	7.1RP	5.7	6.9/3.0
	6/6	2.1RP	3.3	5.8/5.4	1.9RP	1.8	6.2/6.4	6.0RP	3.8	6.3/7.1
	7/2	1.1RP	5.6	6.9/2.1	5.7RP	5.7	7.2/2.6	2.3RP	8.5	7.6/3.1
	7/6	2.9RP	3.1	6.9/6.6	5.0RP	2.3	6.9/7.6	6.3RP	3.8	7.2/8.7
	8/2	1.3RP	4.5	8.2/2.1	7.1RP	4.5	8.1/2.6	2.5R	8.3	8.3/3.1
5.0RP	2/2	5.1R	4.1	2.2/3.3	4.1R	4.8	2.3/3.0	7.1R	3.3	3.0/3.4
	2/4	2.1R	3.6	2.2/6.7	3.7R	3.0	2.4/6.7	3.9R	2.2	2.9/6.4
	2/6	2.9R	2.9	2.2/6.9	2.1R	2.8	2.5/6.3	3.8R	2.0	3.0/6.6
	3/2	2.7R	4.4	2.8/2.6	4.0R	3.5	3.2/2.6	6.8R	2.1	3.6/2.7
	3/6	2.8R	2.1	3.1/7.3	3.9R	1.3	3.2/7.3	4.9R	0.7	3.6/6.7
	3/10	3.0R	1.3	3.3/11.3	2.0R	1.8	3.7/10.6	3.6R	1.2	3.9/9.7
	4/2	2.7R	5.3	5.1/2.3	3.1R	4.5	4.2/2.4	7.5R	3.5	4.9/2.6
	4/6	2.7R	2.1	4.0/7.1	2.3R	1.9	4.4/6.7	4.3R	1.1	4.7/6.6
	4/10	1.5R	1.7	4.3/11.3	0.9R	2.2	4.8/12.1	2.7R	0.5	4.9/10.1
	4/12	2.3R	1.7	4.4/12.3	1.1R	1.1	4.6/11.6	2.9R	0.6	4.6/12.3
	5/2	2.4R	6.5	5.0/2.1	2.3R	3.6	5.4/1.9	9.9R	3.8	5.6/2.8
	5/4	3.5R	7.5	5.0/4.4	1.9R	2.4	5.4/4.6	7.0R	1.9	5.6/5.1
	5/6	2.7R	3.1	5.0/6.1	0.6R	2.2	5.4/6.7	5.4R	1.3	5.7/7.4
	5/8	1.5R	2.3	5.2/8.3	1.6R	2.0	5.6/10.1	3.8R	1.1	5.5/9.3
	5/10	1.6R	6.0	5.2/11.4	1.0R	1.2	5.6/11.7	4.0R	1.3	5.9/10.6
	6/2	2.3R	4.1	6.1/2.1	2.4R	3.6	6.3/2.0	1.1YR	4.1	6.8/3.1
	6/6	1.6R	1.7	5.2/7.1	2.5R	2.0	6.3/8.3	5.9R	0.9	6.6/8.1
	6/10	1.3R	1.7	6.3/10.8	1.6R	2.1	6.1/11.1	4.1R	1.1	6.2/10.4
	7/2	1.3R	4.9	7.2/2.2	3.9R	4.0	7.7/3.5	2.1YR	4.5	7.6/4.0
	7/6	4.5R	2.6	7.2/5.1	3.8R	2.4	7.6/7.9	8.5R	2.9	7.3/6.6
	8/2	3.1R	4.5	8.0/2.3	3.9R	3.6	8.3/3.1	3.1YR	4.5	8.1/4.0
	8/6	2.5R	4.2	8.1/5.1	4.0R	2.8	8.2/5.6	0.4YR	3.4	8.0/6.1
N	1/0	0.8BG	29.7	1.0/0.9	10.0YR	8.1	0.9/0.3	0.6BG	28.1	1.0/0.7
	2/0	1.0BG	31.5	1.5/0.5	4.2Y	11.2	1.9/0.4	9.1Y	22.2	2.1/1.1
	3/0	7.0Y	3.5	2.8/0.5	6.9Y	4.8	2.7/0.7	1.5Y	2.9	3.1/1.7
	4/0	1.9Y	5.4	3.7/0.8	4.9Y	1.7	4.1/0.6	2.0Y	9.4	4.6/1.9
	5/0	8.1Y	4.1	4.8/0.7	5.9Y	3.8	5.2/0.6	2.2Y	1.5	5.5/1.8
	6/0	7.9Y	2.0	6.0/0.5	7.3GY	20.6	6.8/0.9	1.3Y	2.8	6.8/2.3
	7/0	9.1Y	4.5	7.0/1.5	5.0Y	3.0	7.7/0.6	1.3Y	2.2	7.6/2.4
	8/0	10.0Y	7.0	7.9/0.4	4.5Y	3.5	8.4/0.6	0.3Y	2.8	8.1/2.8
	9/0	7.1Y	2.1	8.9/0.5	0.8Y	1.2	9.3/0.6	1.0Y	2.5	8.8/3.6

It will be noted that this observer showed a rather surprising ability to duplicate her previous estimates.

Figure 2 shows an analysis of the data of Table I with respect to changes in hue reported for observations with each background, white, gray, and black, separately. Figure 3 shows an analysis of the same data with the averages taken separately according to the Munsell value (2/, 5/, 8/) of the chip observed. Figure 4 shows a similar analysis with the averages taken separately according to the Munsell chroma (/2, /6, /10) of the chip observed.

IV. DISCUSSION

It will be noted from Fig. 1 (left half) that the changes in color are qualitatively very similar for all three backgrounds, and for light, medium, and dark samples but differ in degree. Samples having purple and red-purple hues exhibit relatively large shifts toward red on being taken from daylight into lamp light. The greenish hues show regular, though smaller hue changes toward reddish yellow. The greatest shifts in hue and chroma are seen to occur on the black background which enhances the hue of the illuminant to a greater degree than either the white or the gray. Furthermore, differential

TABLE II. Repeated matches made by one observer in Illuminant *C* of 29 Munsell samples viewed in Illuminant *A* and the range of hue matches for 7 observers.

Sample in Illuminant *A*	Observer *H* First match *H*	*V/C*	Second match *H*	*V/C*	Hue range for seven observers
5.0YR 2/2	6.3YR	3/2	5.0YR	2.5/4	2.5YR– 6.3YR
3/2	5.0YR	3/4	4.5YR	3/4	3.8YR– 5.0YR
4/2	5.0YR	4/4	5.0YR	4/4	4.5YR– 7.5YR
4/6	5.0YR	4/10	3.8YR	4/8	2.5YR– 5.0YR
5/2	5.0YR	5/3	5.0YR	5/3.5	4.5YR– 7.5YR
5/6	5.0YR	5/10	3.8YR	5/10	3.0YR– 5.0YR
5/10	5.0YR	5/14	3.8YR	5/12	2.0YR– 5.0YR
6/2	6.3YR	6/4	6.3YR	6/4	5.0YR–10.0YR
6/6	5.0YR	6/8	3.8YR	6/10	3.3YR– 5.0YR
6/10	5.0YR	6/16	3.8YR	6/14	2.3YR– 5.0YR
6/12	5.0YR	6/16	2.5YR	6/14	2.5YR– 5.0YR
7/2	5.0YR	7/4	5.0YR	7/4	5.0YR– 7.5YR
7/6	2.5YR	7/9	2.5YR	7/9	2.5YR– 5.0YR
7/8	5.0YR	7/14	3.8YR	7/12	2.0YR– 5.0YR
7/10	5.0YR	7/14	3.8YR	7/14	2.3YR– 5.0YR
8/2	5.0YR	8/4	5.0YR	8/4	5.0YR– 2.5Y
10.0B 2/2	10.0B	2/1	5.0B	2/1	5.0B – 5.0P
3/2	5.0B	3/0.2	5.0B	3/0.5	5.0BG–10.0B
3/6	5.0B	3/3.5	5.0B	3/3	5.0B –10.0B
3/8	5.0B	3/4	5.0B	3/4	2.5B –10.0B
4/2	5.0B	4/0.2	5.0B	4/1	10.0G –10.0B
4/6	5.0B	4/3	5.0B	3.8/2	2.5B – 7.5B
4/8	5.0B	4/4	2.5B	4/4	2.5B – 7.5B
5/2	5.0B	5.5/0.2	5.0B	5/1	3.8Y – 7.5B
5/6	2.5B	5/2	10.0BG	5/2	10.0BG– 5.0B
6/2	5.0B	6.5/0.2	5.0B	6/1	6.3Y – 5.0B
7/2	5.0B	7/0.5	5.0B	7/1	5.0Y – 5.0B
7/6	5.0B	7/3	5.0B	7/3	7.5G – 5.0B
8/2	5.0B	8/0.1	2.5Y	8/2	1.3Y – 5.0B

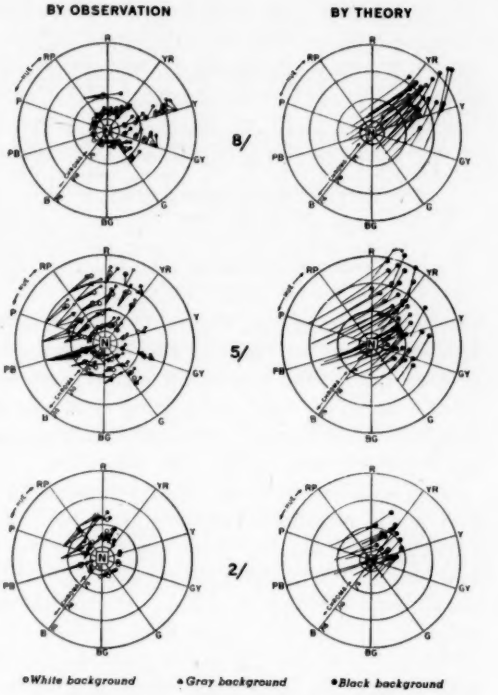

FIG. 1. Hue and chroma of Munsell samples in incandescent-lamp light (Illuminant *A*), as compared with artificial daylight (Illuminant *C*), when observed on white, gray, and black backgrounds, and according to theory on a completely non-reflecting background.

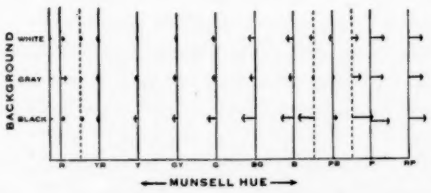

FIG. 2. Average hue shift, regardless of value or chroma, of samples on each of three backgrounds.

effects of background on lightness appear in the fact that on black background maximal shift occurs at the 8/ value level, and on white background at the 2/ value level, while with gray background the changes are about equal at the 8/ and 2/ levels, and smaller at the intermediate values (Fig. 3).

In general the weaker chromas change more than the stronger on all backgrounds as shown by the data plotted in Fig. 4. Changes in chroma

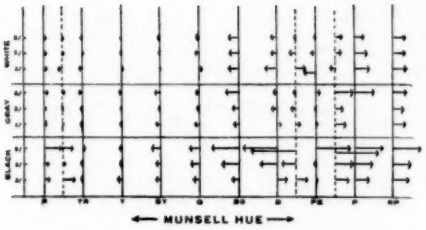

FIG. 3. Average hue shift on white, gray, and black backgrounds at 2/, 5/, and 8/ value levels.

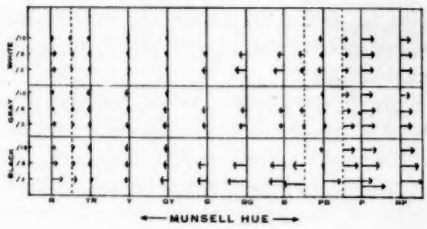

FIG. 4. Average hue shift on white, gray, and black backgrounds at /2, /6 and /10 chroma steps.

also depend upon hue. Figure 1 shows greatest loss in chroma at the 5.0 PB invariant, and greatest gain in the R, and YR region where the second invariant lies.[8] Hues lying between the two invariants lose or gain in saturation as they are nearer the one, or the other invariant.

Two invariant hues appear, one between 10 R and 5 YR on all backgrounds, and the other between 5 PB and 10 PB on white background, and between 10 B and 5 PB on gray and black backgrounds. Reference to Fig. 1 shows that the invariant in the PB region loses in saturation while the one in the YR region gains, so that we may denote the former the negative invariant and the latter positive. Had the matching operation been opposite to what it was in this experiment, i.e., from incandescent-lamp light to daylight, positive and negative invariants would have been reversed. This was found to be the case in a set of observations under such conditions. Similarly the dominant direction of change with matching procedure reversed would be away from incandescent-lamp light toward daylight. That the invariant hues depend to some extent upon reflectance ratio between sample and background is shown by the behavior of 5 YR on black background. The 8/ sample shifts toward yellow, while the 5/ and 2/ samples become redder, indicating that the invariant is yellower at the 8/ level, than at the 5/ and 2/ values.

The qualitative changes can be predicted fairly well from the known I.C.I. specifications of the Munsell papers under Illuminant *A* compared with those for Illuminant *C*.[6] The right half of Fig. 1 shows this comparison. The straight lines start from points corresponding to the Munsell denotation of the chips used.[9] They end at points indicated by solid circles representing the Munsell denotation corresponding to the I.C.I. specifications (Y, x, y) of the same chip computed for Illuminant *A*. These computations apply strictly to an experiment similar to the one carried out, except that the backgrounds in both booths would be such as to reflect no light whatsoever, and the observer would have to be so screened from the respective illuminants that she would have no cue as to which is which. Under these conditions the samples would be expected to resemble spots of light a good deal, and the setting up of a match between them would not differ essentially from the matching of aperture colors. It will be noted that the theoretical hue and saturation changes indicated are very similar, and the invariant hues not far different from those indicated by the actual experiment. The amounts of the change indicated by these computations are, however, much greater than those found in the actual experiment with backgrounds reflecting considerable fractions of the incident light toward the observer.

By the principles of color conversion[3,4] it is to be expected that the observed changes in hue and saturation would increase as the reflectance of the background is lowered; that is, the smallest changes for the samples as a whole would be expected for the white and gray backgrounds, considerably larger for the black background, and much larger for the ideally dark ground, which is referred to by the computed results.

[8] This invariance may be a large factor in accounting for the popularity of colors of 5 YR hue in interior decoration, and for the popularity of colors of 5 PB hue in package design. Similarly, the large hue change caused by taking a purple chip into lamp light may account for the unpopularity of colors of this hue in applications requiring the use of various kinds of light.

[9] S. M. Newhall, D. Nickerson, and D. B. Judd, "Final report of the O. S. A. subcommittee on the spacing of the Munsell colors," J. Opt. Soc. Am. **33**, 385 (1943).

Figure 1, with minor exceptions, bears out this expectation, as does also Fig. 2. It may be concluded that this study supports qualitatively the Helson-Judd principles of color conversion both as to direction of hue and saturation changes, and also as regards the effect of changing the reflectance of the background. Some regularities are evident, however, from Fig. 3, discussed above as change of invariant hues, that apparently cannot be accounted for by these principles; and further exploratory analysis reveals other such regularities. It would seem worth while to compute, by the formulas[4] already derived, the hue, lightness, and saturation to be expected from each sample-background combination studied, and then to make a detailed comparison such as shown in Fig. 1. Since there is doubt, however, regarding to what field the observer in these studies should be considered adapted, application of the formulas is more than a routine computation, and will require some further research. Detailed comparison of these results with theory has to be reserved, therefore, for another communication.

V. ACKNOWLEDGMENTS

The authors are indebted to Miss Dorothy Nickerson for preparation of Figs. 1–4 for publication, including the theoretical data in Fig. 1, and to her and Dr. Deane B. Judd for advice and assistance during the course of the experiment. We also wish to thank Mr. Norman Macbeth, of the Macbeth Corporation, for the lamp installations, and Mrs. Blanche Bellamy, of the Munsell Color Company, for some of the color samples used in this study.

Errata: The Infra-Red Refractive Index and Dispersion of Evaporated Stibnite Films

[J. Opt. Soc. Am. **37**, 119 (1946)]

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THE values for the refractive indices of the mineral stibnite as measured through prisms were reported incorrectly. The correct values of the indices are:

	761 mμ[1]	823 mμ[1]	852 mμ[2]
$X=b=$	3.194	——	——
$Y=a=$	4.046	3.919	3.875
$Z=c=$	4.303	4.193	4.137

[1] A. Hutchinson, Min. Mag. **14**, 199 (1907).
[2] R. Bailly, Bull. classe sci. Acad. roy. Belg. **24**, 791 (1938).

Silberstein on Intrinsic Properties of the Color Domain

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Starting with an elementary consideration of two equally bright colors just distinguishable as to chromaticity, and introducing Fechner's law, Silberstein has developed expressions intended to define the properties of a "color domain obeying Fechner's law." It is shown, however, that Silberstein's method of introducing Fechner's law is inconsistent with his initial assumptions, and that the resulting expressions disagree fundamentally with experimental data.

IN his "Investigations on the Intrinsic Properties of the Color Domain"[1] Silberstein develops expressions purportedly defining the properties of a "color domain obeying Fechner's law." However, a serious inconsistency occurs at the point in the derivation where Fechner's law is introduced. The inconsistency appears to originate through a confusion in the definition of the quantity $\tilde{\omega}_1$. At this place[2] $\tilde{\omega}_1$ is defined as "the least perceptible addition" to the amount x of a given color L contained in another color P of which xL is one of three components. (Similar definitions apply to analogous quantities $\tilde{\omega}_2$ and $\tilde{\omega}_3$ corresponding to the other two components of P.) But $\tilde{\omega}_1$ had previously been defined[3] as the fractional amount of L which must be added, *concomitantly with a decrease in P by a fractional amount* $-\tilde{\omega}_1$, so as to produce a just perceptibly different color R of the same brightness as P. This definition of $\tilde{\omega}_1$ is by no means the same as the other, and the two definitions are irreconcilable.

Utilizing his definition No. 2 of $\tilde{\omega}_1$ (i.e., that it is the least perceptible addition to x), Silberstein finds that by Fechner's law

$$\tilde{\omega}_1 = \alpha_1 x \qquad (1)$$

(or $\tilde{\omega}_1 = \alpha_1 x(1-\tilde{\omega}_1)$; the two equations are sensibly identical) where, as above, x is the amount of L in P, and α_1 is a constant. Silberstein then substitutes this value for $\tilde{\omega}_1$ in his Eqs. (19) *which have been derived on the basis of his definition No. 1 of* $\tilde{\omega}_1$, and thenceforth continues the development of the equations so obtained. This procedure is inadmissible. The derived expressions consequently cannot be said to embrace Fechner's law in the manner claimed by Silberstein.

Silberstein's evaluation of $\tilde{\omega}_1$ (Eq. (1) above) follows directly from definition No. 2 of $\tilde{\omega}_1$, assuming Fechner's law to hold. According to the latter, the increase of stimulus necessary to produce a just perceptible change in sensation is proportional to the stimulus intensity. Here, x is the amount or intensity of the stimulus L, $\tilde{\omega}_1$ is the amount of a just perceptible increase in x, α_1 is the Fechner constant of proportionality.

Equation (1) does not follow from definition No. 1 of $\tilde{\omega}_1$. This becomes apparent upon an extension of the elementary analysis initiated by Silberstein. Employing the usual mathematical and geometrical conventions for the representation of color, Silberstein defines the color P by means of the Maxwellian color-equation

$$P \equiv xL + yM + zN \qquad (2)$$

where L, M, and N are unit amounts of three arbitrarily chosen "primary colors" in the Maxwellian sense. The three primaries are graphically represented as lying at the apices of an equilateral triangle on a plane surface specified by the relation $x+y+z=1$ among the coefficients. The unit amounts of the three primaries are taken equal in brightness, so that the plane of the color-triangle is an "equibright surface," all colors lying on it having the same brightness. The color P is some color lying inside the triangle LMN.

Having thus specified P, Silberstein then considers another color R, just distinguishable from P, lying on the line PL, and denoted by the specification[4] $\tilde{\omega}_1 L + (1-\tilde{\omega}_1)P$. This specification

[1] L. Silberstein, "Investigations on the intrinsic properties of the color domain," J. Opt. Soc. Am. **28**, 63–85 (1938).
[2] Reference 1, top of p. 74.
[3] Reference 1, bottom of p. 73 and Eq. (15).
[4] Reference 1, bottom of p. 73, unnumbered equation.

defines the quantity $\bar{\omega}_1$ in its association with a least perceptible difference of sensation. This is Silberstein's original definition of $\bar{\omega}_1$ (first given in his Eq. (15)), upon the basis of which his Eqs. (19) are derived, and upon the basis of which, therefore, the value of $\bar{\omega}_1$ to be substituted in Eqs. (19) must be determined. The Maxwellian equation for R is thus

$$R \equiv \bar{\omega}_1 L + (1-\bar{\omega}_1)P. \qquad (3)$$

Silberstein's analysis goes no further in this direction. It may, however, be extended by substituting for P in (3) its equivalent in terms of the three primaries from (2):

$$R \equiv \bar{\omega}_1 L + (1-\bar{\omega}_1)(xL + yM + zN),$$

which reduces to

$$R \equiv [x + \bar{\omega}_1(1-x)]L + (y - \bar{\omega}_1 y)M + (z - \bar{\omega}_1 z)N,$$

or

$$R \equiv P + \bar{\omega}_1[(1-x)L - yM - zN]. \qquad (4)$$

The bracketed portion of (4) shows how R differs from P in terms of amounts of the three primaries. It is evident that in general changes in the amounts of all three primaries occur in passing from P to the just distinguishable color R. This result, the direct consequence of Silberstein's definition No. 1 of $\bar{\omega}_1$ (Eq. (3) above), is inconsistent with his definition No. 2 of $\bar{\omega}_1$ whence Eq. (1) is obtained, for even if Silberstein elects to disregard the possible effect of changes in the other two primaries, the change in the amount of L is not $\bar{\omega}_1$, as assumed by Silberstein to obtain (1), but it is $\bar{\omega}_1(1-x)$, which is not the same at all.

If the change in the amount of L is to be set equal to $\alpha_1 x$, then

$$\bar{\omega}_1(1-x) = \alpha_1 x$$

or

$$\bar{\omega}_1 = \alpha_1 x/(1-x),$$

an expression which is quite distinct from (1).

It can be further shown, however, that in the general case considered by Silberstein no function in a single variable can satisfactorily evaluate $\bar{\omega}_1$. Since the relation $x+y+z=1$ holds, (4) may also be written

$$R - P \equiv \bar{\omega}_1[y(L-M) + z(L-N)], \qquad (5)$$

keeping in mind the meaning of a negative term in the Maxwellian color equation, i.e., the minus sign merely signifies that the term has been transposed from one side of the equation to the other.

The right-hand member of (5) specifies the *chromaticity difference* between colors P and R in terms of differences in the amounts of the three primaries L, M, N. This expression contains three variable quantities, namely, $\bar{\omega}_1$, y, z. An interdependence exists among the values of these three variables and the magnitude of the chromaticity difference between P and R. The coordinates y and z are fixed by the choice of P; $\bar{\omega}_1$ is fixed by the condition that the chromaticity difference between P and R is a just perceptible difference or *unit chromaticity step*. The color P, of course, is any color within the triangle LMN, while R lies on the line PL at a distance from P equivalent to one unit chromaticity step. Hence for every color P there is only one corresponding color R, and hence but one corresponding value of $\bar{\omega}_1$. Since the choice of the primaries L, M, N is arbitrary, that is, they may be any three colors within certain broad restrictions,[5] in the general case $\bar{\omega}_1$ must be supposed to assume a different value for each different color P. For if $\bar{\omega}_1$ retained a constant value throughout certain color sequences in the LMN triangle, that fact would entail some sort of affinity among the three primaries, and if such an affinity exists for some sets of primaries L, M, N, with an unlimited number of sets of primaries L, M, N from which to choose, other sets can be chosen which are not so connected. Therefore, in the general case, $\bar{\omega}_1$ must be a function of both y and z. In other words, it is a function of position on the plane of LMN. It is therefore a function of two independent variables and cannot be expressed as a function of a single variable, as Silberstein has sought to do. Moreover, the form of the function would be likely to differ according to choice of the primaries L, M, N, hence probably no general function can be found to express

[5] Reference 1, p. 68. The selection of the primaries is described in the following passage: "A suitable scheme from the practical point of view would be to select some three boundary colors, say L, M, N, which, by blending, are apt to cover the largest portion of the whole color twofold, *viz.* a certain red, green, and violet . . . ."

$\tilde{\omega}_1$, in which case Silberstein's difficulty cannot be resolved.

But disregarding the process by which Silberstein's basic equation ((3) combined with (1)) was derived and accepting it as it stands, its consequences may readily be compared with experimental results, and that it is completely out of harmony with the latter becomes manifest even without the necessity of a detailed comparison. Let the locations of colors P and R in the triangle LMN be $P(x, y, z)$ and $R(x', y', z')$, and set $\Delta x = x' - x$. From Eq. (4)

$$\Delta x = \tilde{\omega}_1(1-x)$$

and from (1)

$$\tilde{\omega}_1 = \alpha_1 x,$$

so that

$$\alpha_1 = \Delta x/x(1-x). \qquad (6)$$

For any given color P the corresponding color R can be determined experimentally, and having done so, the corresponding value of Δx can be calculated. Substituting this quantity in (6) together with the corresponding value of x, the "constant" α_1 may be evaluated. Values of α_1 based on experimental results may thus be found at different points within the LMN triangle, and the nearness with which the separate calculated values of α_1 agree among themselves will give an indication of how well Silberstein's equation (in which α_1 is taken to be constant on the plane of LMN) conforms to actuality.

Individual observed values of Δx differ somewhat, but all results agree in showing Δx as a finite measurable quantity everywhere within the realm of real colors. Since P is any color within the triangle LMN, consider now the case where P lies very near the boundary line MN, so that x approaches zero. In (6) the denominator becomes vanishingly small while the numerator remains finite; hence in this region the value of α_1 exceeds all bounds. The same is true in the vicinity of the point L where $1-x$ approaches zero. Thus, far from being constant, α_1 varies between the limits of some value less than unity (calculated from experimental results near the center of the LMN triangle) and ∞. Moreover, the point L and the line MN near which α_1 approaches ∞, be it remembered, are *arbitrarily located* within the realm of real colors. These considerations alone suffice to make Silberstein's expression quite untenable.

The element of fallacy introduced with the evaluation of $\tilde{\omega}_1$, etc., unfortunately runs through the greater part of Silberstein's rather extended developments beyond that point. The affected portions include, in the section headed "IV. Geometrical Properties of a Color Domain obeying Fechner's Law," under subheading "A. The twofold of colors," Eqs. (21), (22), (22a), (23), (25)–(27), and the series of unnumbered equations on pages 76 and 77 relating to the Gaussian curvature of the twofold, and under subheading "B. The threefold of colors," Eqs. (33), (35), (36), (36a), (38)–(41). These and various other unnumbered equations and intermediate steps in the derivations, as well as Silberstein's inferences drawn therefrom concerning the (non-Euclidean) character of the color domain,[6] are subject to criticism on the following grounds:

1. The final expressions do not actually incorporate Fechner's law in the manner indicated by Silberstein, because of an inconsistency in their derivation.

2. The key quantities $\tilde{\omega}_1$, $\tilde{\omega}_2$, $\tilde{\omega}_3$ are assumed to be definite functions in a single variable whereas in fact they are (probably indefinable) functions in two independent variables.

3. The consequences of Silberstein's basic equation are completely at variance with experimental results.

[6] This criticism does not apply, of course, to conclusions concerning the character of the color domain more recently reached by Silberstein from a different approach.

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A Sensitive Photoelectric Method for Determining the Chromaticity of Phosphorescent and Fluorescent Materials

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(Received February 10, 1947)

A multiplier photo-tube is used in connection with a constant-deviation monochromator to determine the spectral energy distribution for sources having low brightness. The amplification in the tube itself gives high sensitivity without the use of multi-stage amplifier circuits, and slit widths can be made narrow enough to give accurate results for spectral distribution curves generally encountered. The spectral characteristics of two samples of fluorescent cloth are determined first, by using the photoelectric method and, secondly, by the use of a spectrograph with panchromatic plates. A comparison of the two methods is made, and the results are shown graphically by plotting the spectral distribution curves obtained in each case. The convenience, and sensitivity of the photoelectric method gives a marked advantage over the spectrographic method which requires long photographic exposures, and consequent density evaluations.

INTRODUCTION

EXPERIMENTAL studies with phosphorescent or fluorescent materials generally require some method for measuring the spectral characteristics of luminous sources at low brightness values. The brightness may be so low that the ordinary photoelectric cell with amplifier is not sensitive enough to give satisfactory results. However, photoelectric methods using a monochromator have been used to measure spectral energies of luminous materials at comparatively high brightness levels[1] and many of the difficulties of photographic measurements are avoided.

The most dependable methods for finding spectral distribution curves for low luminance materials have involved spectrographic measurements using photography, and some standard source of known energy characteristics.[2] The conversion of densities to energies from wave-length to wave-length is always required, and some uncertainty is introduced where the density values are low. Fortunately, in the determination of the trichromatic coefficients from the spectral energy curve such errors tend to be minimized, but time of exposure and density determinations are somewhat lengthy.

With the development of the multiplier photo-tube, the sensitivity of apparatus involving photoelectric detectors has been greatly increased without the use of complicated amplifier circuits, either a.c. or d.c. Such a tube may be used with a monochromator so that microammeter deflections are obtained with little additional amplification, even for narrow slits and low intensity sources. If the multiplier tube is used in a circuit designed to utilize its maximum sensitivity, spectral energies may be detected which require long exposures with photographic plates.

In order to compare the photoelectric with the photographic method, two samples of fluorescent

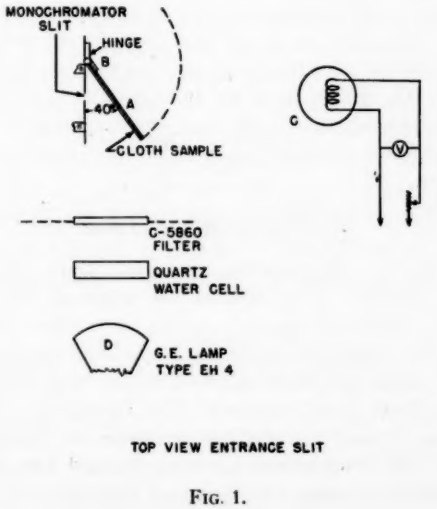

FIG. 1.

[1] J. H. Schulman, "Luminescence of $(Zn, Be)_2SiO_4$:Mn and Other Manganese-Activated Phosphors," J. App. Phys. **17**, 902 (1946).
[2] W. H. Byler and C. C. Carroll, "A Method for Determining the Chromaticity of Fluorescent Material," J. Opt. Soc. Am. **35**, 258 (1945).

cloth were selected, the spectral characteristics of each determined both ways, and a comparison made. One of the samples had a fluorescent color in the yellowish-green region, and the other in the orange. The luminosities of the two samples were relatively high when subjected to strong ultraviolet irradiation which gave shorter exposure times for the method using photographic plates. Much lower values of brightness, however,

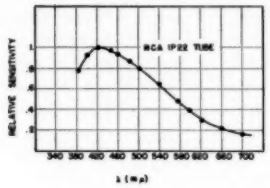

FIG. 2.

could have been measured with the multiplier tube if necessary.

In using the monochromator or spectrograph care was used to obtain identical illumination of the entrance slits by the cloth samples, as illustrated schematically in Fig. 1. The sample was fastened to a blackened piece of metal *A* which had a hinge *B* along its edge. This hinge was soldered to a thin brass strip, machined to slide into the regular grooves of the monochromator head. A tungsten lamp *C* with switch was placed several feet in front of the slit, and light from a mercury lamp *D* was filtered, and introduced from the side so that the ultraviolet rays were approximately parallel to the plane of the slit. An angle of 40° between the cloth and monochromator head was chosen because of optimum slit illumination by the fluorescent cloth.

PHOTOELECTRIC METHOD

In the photoelectric method, the general arrangement of apparatus was centered around a Gaertner constant-deviation monochromator having high light-gathering power. The entrance slit could be illuminated either by fluorescent light from the sample, or by light from a tungsten lamp having a color temperature of 2860°K. Also the monochromator was checked for calibration by using a low pressure Hg arc.

The multiplier tube was placed about four or five centimeters from the exit slit in a light-tight box attached to the monochromator. A shutter arrangement enabled the operator to darken the photo-tube whenever desired. The tube itself was a RCA 1P22 especially chosen for high sensitivity. It was incorporated in a circuit with about 100 volts on each dynode, and the output current fed into the grid circuit of a 6SJ7 tube, which was part of a typical electronic bridge circuit with a microammeter balanced at zero for null conditions. At a given wave-length, a linear relation existed between the light intensity on the photo-cathode of the 1P22, and the deflections of the meter.

The spectral characteristics of the 1P22 multiplier photo-tube are shown in Fig. 2, and are such that measurements can be made over the entire visible region with a high degree of accuracy. Since the tube is not equally sensitive at all wave-lengths in the visible range, a set of correction terms must be used. If the photo-tube and monochromator are considered jointly a calibration can be made by use of the color temperature lamp. If D_λ' is the deflection for the lamp at a wave-length λ, and J_λ is the corresponding energy term from the known values for tungsten at a given color temperature, the correction factor for any wave-length is J_λ/D_λ'. The dispersion term for the monochromator prism cancels out.

In this experiment the samples were bright enough to permit the use of comparatively narrow slits and no slit-width corrections were necessary. Slit widths of 0.04 mm were used and full-scale deflection was obtained for the fluorescent material. Deflections D_λ were recorded for about fifteen wave-length values, the distribution of points being determined by the color of the cloth while being irradiated. For any wave-length λ,

$$E_\lambda = \frac{J_\lambda}{D_\lambda'} D_\lambda,$$

where E_λ represents the relative energy at wave-length λ. While determining the values of the D_λ's, care was used to balance out the dark current.

The spectral energy curves are shown in Figs.

3 and 4 for the two samples of cloth when they are made luminous by ultraviolet light. Solid lines represent the results obtained with the photo-tube, and the dashed lines show the results from the photographic method using a spectrograph. Much care was used to get a comparison of the methods and some details of the second procedure are perhaps worth while.

SPECTROGRAPHIC METHOD

A Bausch and Lomb spectrograph with Eastman Type III F plates was used in the second method. A low pressure Hg arc was used for wave-length measurements, and density-energy studies made with reference to a tungsten lamp at color temperature 2860°K. The arrangement at the entrance slit was essentially the same as that shown in Fig. 1, with the addition of a photographic shutter and lens placed in the optical path between the lamp and entrance slit. A set of screens of known transmission values, placed in front of the lens, gave a series of exposures on the plate for equal shutter speeds, and by evaluating these densities the relative energy values of the unknown were determined at fourteen wave-lengths. The curves representing a relation between densities and energies of the tungsten lamp are shown in Fig. 5. Each curve was determined by six points obtained from six exposures to the lamp.

An exposure of approximately five minutes was sufficient for the fluorescent light given off by the cloth samples. A density curve was obtained, and relative energies E read graphically from the composite set of curves described above.

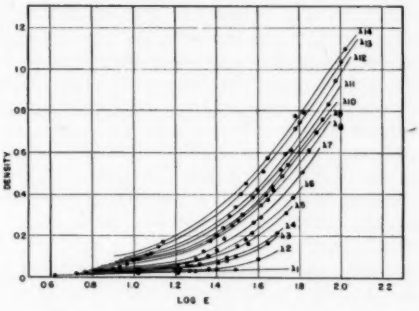

FIG. 5.

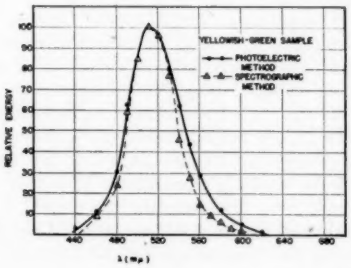

FIG. 3.

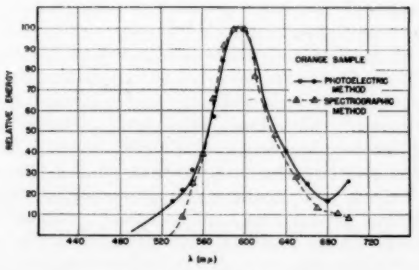

FIG. 4.

Any variation in spectral sensitivity of the plate is accounted for, and since the difference in time of exposure of the sample and the color temperature lamp was less than five minutes, the reciprocity law was assumed to hold. However, the accuracy of the Hurter and Driffield curves at points near fog densities was not high, and a slight failure of the reciprocity law was known to exist. Consequently, as seen in Figs. 3 and 4, there was a slight difference between the two sets of curves and the result was about the same for several other samples. Such a discrepancy may be considered slight for many purposes, and the dominant wave-length and excitation purity of each sample as computed from results of the two methods, were practically the same.

CONCLUSION

While it has been shown that the color characteristics of phosphorescent and fluorescent material may be obtained either by the photoelectric or photographic method with about the same degree of accuracy, the speed and ease of

manipulation of the method using a multiplier photo-tube make it more desirable. By selecting a tube especially for sensitivity, and using it to fullest advantage, reliable meter deflections may be obtained for radiant energies that would require quite long exposures on a photographic plate.

ACKNOWLEDGMENTS

An appreciation is expressed by the author for the helpful suggestions of Dr. L. H. Dawson, Head of the Photometry Section, Optics Division, Naval Research Laboratory, and for the assistance in experimental work and computations given by Mr. L. R. Furlong.

Changes in Photometric Units

In pursuance of decisions of the International Committee on Weights and Measures, the National Bureau of Standards will introduce as of January 1, 1948, revised values of the units of electricity and of light. While the definitions of the units and the methods of fixing their magnitudes will be different from the present practical systems, the changes in magnitude will be so small as to affect appreciably only measurements of high precision. In certificates for standards and instruments issued by the Bureau during 1947 values will be given in both the old and the new units.

The new system of photometric units takes as the primary standard a blackbody radiator operated at the temperature of freezing platinum.[1] The "candle," unit of intensity, is defined as one-sixtieth of the intensity of one square centimeter of such a radiator. Other units are derived from the basic candle, with the provision that when differences of color are involved the evaluation shall be made by means of standard spectral luminosity factors[2] which have been adopted by the International Commission on Illumination and the International Committee on Weights and Measures.

Units of light have never been established by law in the United States. In 1909 the National Bureau of Standards by agreement with laboratories in France and Great Britain introduced the "international candle"; the agreement, however, did not provide for any primary reference standard. Pending the development of a satisfactory basic standard the "candle" was represented by groups of carbon-filament lamps. As other types of lamps were developed, working standards of the new types were set up independently in various countries. This required visual comparison of lights of different colors, which is very difficult, and led to some diversity of units. Moreover, Germany and Austria never accepted the "international" units of light.

As a result of years of discussion and experimental work on standards of candlepower and on photometry of lights of different colors, it was generally agreed that the best type of primary standard now available is the platinum blackbody radiator, and that lights of different colors can best be evaluated by using luminosity factors representing the spectral sensitivity of the average eye under specified conditions.

These two elements give a basis for a complete system of photometric units when a numerical value is assigned for the brightness of the platinum blackbody standard. Its brightness expressed in international units was experimentally determined to be about 58.9 candles per square centimeter. The adoption of the round value 60 candles per square centimeter therefore makes the new unit about 1.9 percent smaller than the international candle at the color-temperature of the primary standard, and sources of light of such color are correspondingly given higher numerical ratings. However, when sources of higher color-temperature are compared with the basic standards, the standard spectral luminosity factors give slightly lower values for the "whiter" sources than were obtained by the visual measurements by which the present international units were established. The difference between the two scales therefore grows less as the color-temperature of the sources measured is increased, and for sources in the range of ordinary vacuum tungsten-filament lamps, around 2500°K, the new scale crosses the "international" scale as used in the United States. Furthermore, ratings for gas-filled, tungsten-filament lamps and other new types have been made by methods more nearly in accord with the luminosity factors. Consequently the present ratings of tungsten-filament lamps in this country will be practically unaffected by the change of systems, no type being changed by more than one percent.

In European countries somewhat higher values have been assigned for light from the high temperature sources. Consequently use of the new scale will bring about reductions of several percent in their ratings, but these changes will not be so great as would have resulted if the standard luminosity factors had been used to step up from the international candle as represented by carbon-filament lamps.

Names of Units.—During the period of transition between systems, in order to avoid any doubt as to the units used in giving precise data, the International Committee on Weights and Measures has recommended that the adjectives "international" (abbreviation "int") and "new" be used with the names of photometric units. The use of these distinguishing terms will become unnecessary, except in referring to old data, within a few years.

[1] Wensel, Roeser, Barbrow, and Caldwell, "The Waidner-Burgess standard of light," Bur. Stand. J. Research **6**, 1103 (1931), RP 325. G. Ribaud, "Realisation de l'étalon primaire d'intensité lumineuse," Rev. d'Optique **12**, 289 (1933). Report (National Physical Laboratory, Great Britain, 1935), p. 79.

[2] K. S. Gibson and E. P. T. Tyndall, "Visibility of radiant energy," Sci. Pap. Bur. Stand. **19**, 131 (1923), S.P. 475; Trans. I.E.S. **19**, 176 (1924); for adoption, see Recueil des Travaux, Comm. Int. de l'Eclairage, 1924, p. 67 and p. 232.

Letters to the Editor

Notes on the Stability of Phosphors under Ultraviolet Irradiation

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March 12, 1947

THE main issues to be considered are: (a) Is the presence of mercury vapor necessary in the deterioration of a phosphor? (b) What ultraviolet radiations effect deterioration of a phosphor?

We maintain that phosphor deterioration can be effected in the absence of mercury vapor with radiation below 2000A, while Froelich contends in his first paper[1] that the decline of lumen output of phosphors is caused by condensed and adsorbed metallic mercury and mercury compounds but that the phosphor is unchanged by either excited mercury or ultraviolet radiation, and in his second paper[2] he concludes that lamp depreciation is fundamentally of photo-chemical nature involving excited mercury atoms caused by 2537A.

Our paper[3] definitely shows that a well-baked 3500° white phosphor deteriorates when irradiated with a quartz mercury lamp in the absence of mercury, but Froelich maintains that an unbaked phosphor contains enough adsorbed gases to cause the reported losses in brightness. Therefore tests were made, and they revealed that on well-evacuated and well-baked phosphors, and in the absence of mercury vapor, deterioration only occurred when the phosphor was irradiated with a quartz mercury discharge lamp which is comparatively rich in 1849A but not when Vycor mercury discharge lamp was used which transmits practically no 1849A but an abundance of 2537A. The question of minute amounts of oxygen or other gaseous impurities has been anticipated and was investigated. A quartz bulb was half-coated with a normal zinc beryllium silicate. A charcoal trap was attached to the bulb and then both were well exhausted. The bulb was then exposed to a seasoned quartz lamp. During the entire test the charcoal trap was immersed in liquid nitrogen. The phosphor deteriorated in the normal manner indicating that oxygen or other gas impurities were not involved in the loss of output. A similar test using a Vycor bulb did not show deterioration thus conclusively proving that radiations less than 2000A are the chief cause of phosphor deterioration in fluorescent lamps and that the process is photochemical in nature.

It will be recalled from the publication[3] that the purpose of using Sterilamps was to demonstrate that the deterioration of phosphors was proportional to the 1849A output. Since the germicidal lamps produce a great deal of 1849A radiation when they are new, loss of output of the phosphors shown by Froelich in his second paper clearly indicates he was dealing with the short radiations and not 2537A.

Froelich mentions that if short ultraviolet radiation produces photolysis then it should be observed to an even greater extent with argon and neon. The experiments in this field have been very limited. Also, the amount of fluorescence produced by these rare gases is exceedingly small so that experiments would have to be conducted over a very much longer time before definite conclusions can be drawn on this subject. In fact, preliminary experiments here indicated a deterioration with the rare gases.

We know that in the presence of oxygen 1849A produces ozone and also that 2537A destroys it, but since we have observed this photo-chemical effect under good vacuum conditions in the absence of mercury, we still maintain that radiations below 2000A are the chief cause of deterioration and that secondary chemical effect, with or without mercury or oxygen, as expressed by Froelich plays only a minor role.

[1] H. C. Froelich, Trans. Am. Electrochem. Soc. **87**, 365 (1945).
[2] H. C. Froelich, J. App. Phys. **17**, 573 (1946).
[3] G. Meister and R. Nagy, J. Opt. Soc. Am. **36**, 696 (1946).

Mechanical Integrator for Studying the Distribution of Light in the Optical Image

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February 19, 1947

WHEN the geometrical aberrations of an optical instrument have been determined, it is possible to calculate the surface form of a particular wave Σ in the image space. The Huyghens-Fresnel principle then makes possible the calculation of the illumination at a point C in the following manner (see Fig. 1.):

Assuming that S is a sphere with center C, M a point of Σ, I a point where MC intersects S, $IM=\Delta$ is the normal deviation of the wave surface from the sphere S. The components of the amplitude of the vibration are proportional to:

$$A=\int\int_{\tau}\cos\psi d\omega, \quad B=\int\int_{\tau}\sin\psi d\omega, \qquad (1)$$

where τ is the contour of Σ, ψ the phase difference $\psi=2\pi\Delta/\lambda$, and $d\omega$ the surface element of Σ if the total surface of Σ is taken as a unit.

The integrator to be described permits the calculation of A and B, and accordingly, the determination of the intensity and phase of the resultant vibration in case of a rotationally symmetrical instrument possessing a circular exit pupil with radius r.

Let us now transform the double integrals into simple integrals using as variables the polar coordinates ρ and ϕ; with respect to the pupil we have

$$d\omega=\frac{1}{\pi R^2}\rho d\rho d\phi. \qquad (2)$$

Next we describe an Archimedean spiral on that pupil, with a parameter sufficiently small so that we can sub-

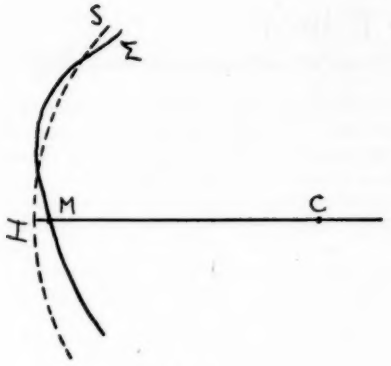

FIG. 1.

stitute the integration with relation to the variable ρ by a finished sum of distant terms of $d\rho=a$. Then we still have to carry out the continuous integration of the quantities $\rho\cos\psi$ and $\rho\sin\psi$ with relation to ϕ as variable, and it is necessary to determine Δ (or ψ) as a function of ϕ.

The path difference, Δ, depends upon the geometric aberrations and the position of point C; if we define C by cylindrical coordinates with the paraxial focus P as origin, with the axis coinciding with a mean ray (the principal ray), one of the coordinates will be the shift of focus with field given by the distance of C from the plane perpendicular to the axis passing through P; the other two coordinates allow C to vary in the paraxial focal plane. It is found that the path difference is composed of five terms. The addition of these terms may be done mechanically by means of a steel tape passing over five pulleys. The first two terms, which correspond to the spherical aberration and the focusing shift (or the curvature of field), depend only upon ρ and may be obtained by means of cams whose displacement represents the variable ρ (which is a linear function of ϕ). The term representing the coma of the form $f(\rho)\cos\phi$ will be obtained by the movement of a crank turning according to angle ϕ, the radius of which is proportional to $af(\rho)$ as determined by a cam. The exploration of curvature and astigmatism are represented, respectively, by the terms of the form $\rho\cos(\phi-\phi_0)$ and $f(\rho^2)\cos2\phi$ and are obtained according to the same procedure. Amplifier levers permit the variation of the relative importance of these five terms. The displacement of the tape represents the path difference Δ, and if the ribbon is wound on a pulley of suitable radius, the rotation of which is equal to the difference of phase ψ, it is easy to perform the integration of the quantities $\rho\cos\psi$ and $\rho\sin\psi$. Therefore two cranks with radius ρ, (obtained by means of a cam) and turning in proportion to the difference of angular phase ψ, push two integration rollers into contact with the plates whose rotation will be given by ψ.

In conclusion we may state that, in order to limit integration to a perfectly circular contour, we must modify slightly the extremities of various cams. (It is possible to study the case of a pupil of any contour by attaching to the apparatus a convenient shifter.)

Accordingly, this machine makes it possible to determine the distribution of the light in a small area surrounding the paraxial focus in the presence of different geometric aberrations, isolated or superimposed. To do this, the optical calculator would install on the apparatus cams representing the results of his calculations. Next the diffraction factor would be determined systematically, each point requiring an operation taking 8 to 10 minutes depending upon the individual case. We have checked the operation of the machine by employing calculations already done by different authors (Airy's work on diffraction; and Nyboer's work on curvature of field, spherical aberration, coma, and astigmatism). The absolute error in the amplitude of vibration generally does not exceed 5×10^{-3} which corresponds to an error of intensity usually less than the following values:

$$\Delta I<0.01 \quad \Delta I<0.003 \quad \Delta I<0.001$$
$$\text{if } I\fallingdotseq 1, \quad \text{if } I\fallingdotseq 0.1, \quad \text{if } I\fallingdotseq 0.01, \quad \text{etc.}$$

The precision of the apparatus is accordingly entirely sufficient for practical purposes.

* Translated by Max Herzberger, Eastman Kodak Company, Rochester, New York.